# Cryptographic Hash Function 

## Blue Midnight Wish

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September 2009

## Abstract

This is the supporting documentation that describes in details the tweaked cryptographic hash function Blue Midnight Wish which is submitted as a candidate for the second round of the SHA-3 hash competition organized by the National Institute of Standards and Technology (NIST), according to the public call [1].

BLUE MIDNIGHT WISH is a cryptographic hash function with output size of $n$ bits where $n=224$, 256,384 or 512 . Its conjectured cryptographic security is: $O\left(2^{\frac{n}{2}}\right)$ hash computations for finding collisions, $O\left(2^{n}\right)$ hash computations for finding preimages, $O\left(2^{n-k}\right)$ hash computations for finding second preimages for messages shorter than $2^{k}$ bits. Additionally, it is resistant against lengthextension attacks, and it is resistant against multicollision attacks.

Blue Midnight Wish has been designed to be much more efficient than SHA-2 cryptographic hash functions, while in the same time offering same or better security. The speed of the optimized 32-bit version on the defined reference platform using $\operatorname{Intel}(\mathrm{R}) \mathrm{C}++11.0 .072$ is 7.76 cycles/byte for $n=224,256$ and 13.20 cycles/byte for $n=384,512$. The speed of the optimized 64 -bit version on the defined reference platform using $\operatorname{Intel}(\mathrm{R}) \mathrm{C}++11.0 .072$ is 7.50 cycles / byte for $n=224,256$ and 3.90 cycles/byte for $n=384,512$.

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## CHAPTER 1

## Algorithm Specifics

### 1.1 Bit Strings and Integers

The following terminology related to bit strings, byte strings and integers will be used:

1. A hex digit is an element of the set $\{0,1, \ldots, 9, A, \ldots, F\}$. A hex digit is the representation of a 4-bit string. For example, the hex digit " 7 " represents the 4-bit string " 0111 ", and the hex digit " A " represents the 4-bit string "1010".
2. The "little-endian" convention is used when expressing string of bytes stored in memory. That means that beginning from some address " H " if the content of the memory is represented as a 1-byte address increment, then 32-bit and 64-bit integers are expressed as in the example given in Table 1.1. The prefix " 0 x " is used to annotate that the integer is expressed in hex digit notation.
3. The "big-endian" convention is used when expressing the "internal bit endianness" for both 32 -bit and 64 -bit words as integers. That means that within each word, the most significant bit is stored in the left-most bit position. More concretely, a word is a $w$-bit string that may be represented as a sequence of hex digits. To convert a word to hex digits, each 4-bit string is converted to its hex digit equivalent. For example, the 32-bit string "1010 0001000000111111111000100011 " has a hexadecimal representation "0xA103FE23" and its value as unsigned long integer is 2701393443 . The 64-bit string "1010 0001000000111111 1110001000110011001011101111001100000001 1010" has a hexadecimal representation "0xA103FE2332EF301A" and its value as unsigned long long integer is 11602396492168376346 .
4. For Blue Midnight Wish hash algorithm, the size of $m$ bits of the message block, depends

| Address in memory | Byte value |
| :---: | :---: |
| H | 23 |
| $\mathrm{H}+1$ | FE |
| $\mathrm{H}+2$ | 03 |
| $\mathrm{H}+3$ | A 1 |


| Address in memory | Byte value |
| :---: | :---: |
| H | 1 A |
| $\mathrm{H}+1$ | 30 |
| $\mathrm{H}+2$ | EF |
| $\mathrm{H}+3$ | 32 |
| $\mathrm{H}+4$ | 23 |
| $\mathrm{H}+5$ | FE |
| $\mathrm{H}+6$ | 03 |
| $\mathrm{H}+7$ | A 1 |

64-bit integer value: "0xA103FE2332EF301A"
Table 1.1: Default design of the Blue Midnight Wish is "Little-endian" on the variant of algorithm (BMW224, BMW256, BMW384 or BMW512).
(a) For BMW224 and BMW256, each message block has 512 bits, which are represented as a sequence of sixteen 32-bit words.
(b) For BMW384 and BMW512, each message block has 1024 bits, which are represented as a sequence of sixteen 64-bit words.

### 1.2 Parameters, variables and constants

The following parameters and variables are used in the specification of Blue Midnight Wish:

H
$Q$
$H^{(i)}$
$Q^{(i)}$
$H_{j}^{(i)}$

Double pipe. It is a chaining value that is at minimum two times wider than the final message digest of $n$ bits.

Quadruple pipe.
The $i$-th double pipe value. $H^{(0)}$ is the initial double pipe value. $H^{(N)}$ is the final double pipe value and is used to determine the message digest of $n$ bits.

The $i$-th quadruple pipe value.
The $j$-th word of the $i$-th double pipe value $H^{(i)}$, where $H_{0}^{(i)}$ is the is the left-most word.
$K_{j}=j \times(0 \times 05555555)$
$j=16,17, \ldots, 31$

0x0555555555555555

The $j$-th word of the $i$-th quadruple pipe value $Q^{(i)}=$ $\left(Q_{0}^{(i)}, \ldots, Q_{31}^{(i)}\right)$, where $Q_{0}^{(i)}$ is the left-most word.

The first 16 words from $Q^{(i)}$, i.e. $Q_{a}^{(i)}=\left(Q_{0}^{(i)}, \ldots, Q_{15}^{(i)}\right)$.
The last 16 words from $Q^{(i)}$, i.e. $Q_{b}^{(i)}=\left(Q_{16}^{(i)}, \ldots, Q_{31}^{(i)}\right)$.
Number of zeroes appended to a message during the padding step.

Length of the message $M$, in bits.
Number of bits in a message block, $M^{(i)}$.
Message to be hashed.
Message block $i$, with a size of $m$ bits.

The $j$-th word of the $i$-th message block $M^{(i)}=\left(M_{0}^{(i)}, \ldots, M_{15}^{(i)}\right)$, where $M_{0}^{(i)}$ is the is the left-most word.

Number of bits to be rotated or shifted when a word is operated upon.

Number of blocks in the padded message.
Two temporary words (32-bit or 64-bit - depending on the variant of the algorithm ) used in the computation of the double pipe.

A hex digit representation of a 32 -bit constant (unsigned long integer).

A 32-bit constant (unsigned long) obtained by multiplying the constant $0 \times 05555555$ by an integer $j$, where $j$ is in the range from 16 to 31 .

A hex digit representation of a 64-bit constant (unsigned long long integer).

$$
\begin{aligned}
& K_{j}=j \times(0 \times 0555555555555555) \\
& j=16,17, \ldots, 31
\end{aligned}
$$

ExpandRounds ${ }_{1}=2$,
ExpandRounds ${ }_{2}=14$
CONST ${ }^{\text {final }}$ for BMW224/256:
CONST:

|  | 0xaaaaaaac, 0xaaaaaaad, 0xaaaaaaae, 0xaaaaaai) |
| :---: | :---: |
|  |  |
|  | 0xaaaaaaaaaaaaa33, 0xaaaaaaaaaaaaaa, 0xaaaaaaaaaaaaa5, |
| CONST ${ }^{\text {final }}$ for BMW384/512: | 0xaaaaaaaaaaaaaa6, 0xaaaaaaaaaaaaa7, 0xaaaaaaaaaaaaa8, |
|  | 0xaaaaaaaaaaaaa.9, 0xaaaaaaaaaaaaaa, 0xaaaaaaaaaaaaaab, |
|  | 0xaaaaaaaaaaaaac, 0xaaaaaaaaaaaaad, 0xaaaaaaaaaaaaae |
|  |  |

### 1.3 General design properties of Blue Midnight Wish

Blue Midnight Wish follows the general design pattern that is common for most known hash functions. It means that it has two stages (and several sub-stages within every stage):

1. Preprocessing
(a) padding a message,
(b) parsing the padded message into $m$-bit blocks, and
(c) setting initialization values to be used in the hash computation.
2. Hash computation
(a) generating a message schedule from the padded message,
$\begin{array}{|c|c|c|c|c|c|c|}\hline \text { Algorithm } \\ \text { abbreviation }\end{array} \begin{array}{c}\text { Message size } \\ l \text { (in bits) }\end{array} \quad \begin{array}{c}\text { Block size } \\ m \text { (in bits) }\end{array} \quad \begin{array}{c}\text { Word size } \\ w \text { (in bits) }\end{array} \quad$ Endianess $\left.\begin{array}{c}\text { Support of } \\ n \text { (in bits) }\end{array} \begin{array}{c}\text { "one-pass" } \\ \text { streaming } \\ \text { mode }\end{array}\right]$

Table 1.2: Basic properties of all four variants of the BLUE Midnight Wish
(b) using that schedule, along with functions, constants, and word operations to iteratively generate a series of double pipe values,
(c) the final double pipe value generated by the iterative process in (b) is used as an input value for a finalization function (which is essentially the same compression function but with different inputs and constants),
(d) the $n$ Least Significant Bits (LSB) of the finalization function are used to determine the message digest.

Depending on the context we will sometimes refer to the hash function as BLUE MIDNIGHT Wish and sometimes as BMW224, BMW256, BMW384 or BMW512.

In Table 1.2, we give the basic properties of all four variants of the Blue Midnight Wish hash algorithms.

The following operations are applied in BLUE Midnight Wish:

1. Bitwise logic word operations $\oplus-X O R$.
2. Addition + and subtraction - modulo $2^{32}$ or modulo $2^{64}$.
3. Shift right operation, $S \operatorname{SR}^{r}(x)$, where $x$ is a 32 -bit or 64 -bit word and $r$ is an integer with $0<r<32$ (resp. $0<r<64$ ).
4. Shift left operation, $S H L^{r}(x)$, where $x$ is a 32 -bit or 64 -bit word and $r$ is an integer with $0<r<32$ (resp. $0<r<64$ ) .
5. Rotate left (circular left shift) operation, $\operatorname{ROTL}^{r}(x)$, where $x$ is a 32 -bit or 64 -bit word and $r$ is an integer with $0<r<32$ (resp. $0<r<64$ ).

### 1.4 Blue Midnight Wish logic functions

Blue Midnight Wish uses the logic functions, summarized in Table 1.3.

| BMW224/BMW256 | BMW384/BMW512 |
| :---: | :---: |
| $s_{0}(x)=S H R^{1}(x) \oplus \operatorname{SHL}^{3}(x) \oplus \operatorname{ROTL}^{4}(x) \oplus \operatorname{ROTL}^{19}(x)$ | $s_{0}(x)=S H R^{1}(x) \oplus \operatorname{SHL}^{3}(x) \oplus \operatorname{ROTL}^{4}(x) \oplus \operatorname{ROTL}^{37}(x)$ |
| $s_{1}(x)=S H R^{1}(x) \oplus \operatorname{SHL}^{2}(x) \oplus \operatorname{ROTL}^{8}(x) \oplus \operatorname{ROTL}^{23}(x)$ | $s_{1}(x)=S H R^{1}(x) \oplus \operatorname{SHL}^{2}(x) \oplus \operatorname{ROTL}^{13}(x) \oplus \operatorname{ROTL}^{43}(x)$ |
| $s_{2}(x)=S H R^{2}(x) \oplus \operatorname{SHL}^{1}(x) \oplus \operatorname{ROTL}^{12}(x) \oplus \operatorname{ROTL}^{25}(x)$ | $s_{2}(x)=\operatorname{SHR}^{2}(x) \oplus \operatorname{SHL}^{1}(x) \oplus \operatorname{ROTL}^{19}(x) \oplus \operatorname{ROTL}^{53}(x)$ |
| $s_{3}(x)=S H R^{2}(x) \oplus \operatorname{SHL}^{2}(x) \oplus \operatorname{ROTL}^{15}(x) \oplus \operatorname{ROTL}^{29}(x)$ | $s_{3}(x)=S H R^{2}(x) \oplus \operatorname{SHL}^{2}(x) \oplus \operatorname{ROTL}^{28}(x) \oplus \operatorname{ROTL}^{59}(x)$ |
| $s_{4}(x)=\operatorname{SHR}^{1}(x) \oplus x$ | $s_{4}(x)=\operatorname{SHR}^{1}(x) \oplus x$ |
| $s_{5}(x)=\operatorname{SHR}^{2}(x) \oplus x$ | $s_{5}(x)=\operatorname{SHR}^{2}(x) \oplus x$ |
| $r_{1}(x)=\operatorname{ROTL}^{3}(x)$ | $r_{1}(x)=\operatorname{ROTL}^{5}(x)$ |
| $r_{2}(x)=\operatorname{RotL}^{7}(x)$ | $r_{2}(x)=\operatorname{ROTL}^{11}(x)$ |
| $r_{3}(x)=\operatorname{ROTL}^{13}(x)$ | $r_{3}(x)=\operatorname{ROTL}^{27}(x)$ |
| $r_{4}(x)=\operatorname{ROTL}^{16}(x)$ | $r_{4}(x)=\operatorname{ROTL}^{32}(x)$ |
| $r_{5}(x)=\operatorname{ROTL}^{19}(x)$ | $r_{5}(x)=\operatorname{ROTL}^{37}(x)$ |
| $r_{6}(x)=\operatorname{ROTL}^{23}(x)$ | $r_{6}(x)=\operatorname{ROTL}^{43}(x)$ |
| $r_{7}(x)=\operatorname{ROTL}^{27}(x)$ | $r_{7}(x)=\operatorname{ROTL}^{53}(x)$ |
| $\text { AddElement }(j)=\left(\operatorname{ROTL} L^{(j+1)}\left(M_{j}^{(i)}\right)+\operatorname{ROTL}^{(j+4)}\left(M_{j+3}^{(i)}\right)\right.$ | AddElement $(j)=\left(\operatorname{ROTL}^{(j+1)}\left(M_{j}^{(i)}\right)+\operatorname{ROTL}^{(j+4)}\left(M_{j+3}^{(i)}\right)\right.$ |
| $\left.-\operatorname{ROTL}^{(j+11)}\left(M_{j+10}^{(i)}\right)+K_{j+16}\right) \oplus H_{j+7}^{(i)}$ | $\left.-\operatorname{ROTL}^{(j+11)}\left(M_{j+10}^{(i)}\right)+K_{j+16}\right) \oplus H_{j+7}^{(i)}$ |
| $\operatorname{expand}_{1}(j)=s_{1}\left(Q_{j-16}^{(i)}\right)+s_{2}\left(Q_{j-15}^{(i)}\right)+s_{3}\left(Q_{j-14}^{(i)}\right)+s_{0}\left(Q_{j-13}^{(i)}\right)$ | $\operatorname{expand}_{1}(j)=s_{1}\left(Q_{j-16}^{(i)}\right) \quad+s_{2}\left(Q_{j-15}^{(i)}\right)+s_{3}\left(Q_{j-14}^{(i)}\right)+s_{0}\left(Q_{j-13}^{(i)}\right)$ |
| $+s_{1}\left(Q_{j-12}^{(i)}\right) \quad+s_{2}\left(Q_{j-11}^{(i)}\right)+s_{3}\left(Q_{j-10}^{(i)}\right)+s_{0}\left(Q_{j-9}^{(i)}\right)$ | $+s_{1}\left(Q_{j-12}^{(i)}\right) \quad+s_{2}\left(Q_{j-11}^{(i)}\right)+s_{3}\left(Q_{j-10}^{(i)}\right)+s_{0}\left(Q_{j-9}^{(i)}\right)$ |
| $+s_{1}\left(Q_{j-8}^{(i)}\right)+s_{2}\left(Q_{j-7}^{(i)}\right)+s_{3}\left(Q_{j-6}^{(i)}\right)+s_{0}\left(Q_{j-5}^{(i)}\right)$ | $+s_{1}\left(Q_{j-8}^{(i)}\right)+s_{2}\left(Q_{j-7}^{(i)}\right)+s_{3}\left(Q_{j-6}^{(i)}\right)+s_{0}\left(Q_{j-5}^{(i)}\right)$ |
| $\begin{aligned} & +\underset{s_{1}\left(Q_{j-4}^{(i)}\right)}{+}+s_{2}\left(Q_{j-3}^{(i)}\right)+s_{3}\left(Q_{j-2}^{(i)}\right)+s_{0}\left(Q_{j-1}^{(i)}\right) \\ & + \text { AddElement }(j-16) \end{aligned}$ | $\begin{aligned} & +\underset{s_{1}\left(Q_{j-4}^{(i)}\right)}{+}+s_{2}\left(Q_{j-3}^{(i)}\right)+s_{3}\left(Q_{j-2}^{(i)}\right)+s_{0}\left(Q_{j-1}^{(i)}\right) \\ & + \text { AddElement }(j-16) \end{aligned}$ |
| $\operatorname{expand}_{2}(j)=\quad Q_{j-16}^{(i)}{ }^{\text {a }}$ ( $r_{1}\left(Q_{j-15}^{(i)}\right)+Q_{j-14}^{(i)}+r_{2}\left(Q_{j-13}^{(i)}\right)$ | $\operatorname{expand}_{2}(j)=\quad Q_{j-16}^{(i)}+r_{1}\left(Q_{j-15}^{(i)}\right)+Q_{j-14}^{(i)}+r_{2}\left(Q_{j-13}^{(i)}\right)$ |
| $+\quad Q_{j-12}^{(i)}+r_{3}\left(Q_{j-11}^{(i)}\right)+Q_{j-10}^{(i)}+r_{4}\left(Q_{j-9}^{(i)}\right)$ | $+\quad Q_{j-12}^{(i)}+r_{3}\left(Q_{j-11}^{(i)}\right)+Q_{j-10}^{(i)}+r_{4}\left(Q_{j-9}^{(i)}\right)$ |
| $+\quad Q_{j-8}^{(i)}+r_{5}\left(Q_{j-7}^{(i)}\right)+Q_{j-6}^{(i)}+r_{6}\left(Q_{j-5}^{(i)}\right)$ | $+\quad Q_{j-8}^{(i)}+r_{5}\left(Q_{j-7}^{(i)}\right)+Q_{j-6}^{(i)}+r_{6}\left(Q_{j-5}^{(i)}\right)$ |
| $+\quad Q_{j-4}^{(i)}+r_{7}\left(Q_{j-3}^{(i)}\right)+s_{4}\left(Q_{j-2}^{(i)}\right)+s_{5}\left(Q_{j-1}^{(i)}\right)$ | $+\quad Q_{j-4}^{(i)}+r_{7}\left(Q_{j-3}^{(i)}\right)+s_{4}\left(Q_{j-2}^{(i)}\right)+s_{5}\left(Q_{j-1}^{(i)}\right)$ |
| + AddElement $(j-16)$ | + AddElement $(j-16)$ |

Table 1.3: Logic functions used in Blue Midnight Wish. Note that for the function AddElement $(j)$ index expressions involving the variable $j$ for left rotations, $M$ and $H$ are computed modulo 16.

### 1.5 Preprocessing

Preprocessing consists of three steps:

1. padding the message M ,
2. parsing the padded message into message blocks, and
3. setting the initial double pipe value, $H^{(0)}$.

### 1.5.1 Padding the message

The message $M$, shall be padded before hash computation begins. The purpose of this padding is to ensure that the padded message is a multiple of 512 or 1024 bits, depending on the size of the message digest $n$.

## BWM224 and BMW256

Suppose that the length of the message $M$ is $l$ bits. Append the bit " 1 " to the end of the message, followed by $k$ zero bits, where $k$ is the smallest, non-negative solution to the equation $l+1+k \equiv$ $448 \bmod 512$. Then append the 64 -bit block that is equal to the number $l$ expressed using its littleendian binary representation. For example, the message "abc" encoded in 8-bit ASCII has length $8 \times 3=24$, so the message is padded with the bit " 1 ", then $448-(24+1)=423$ zero bits, and then the 64-bit binary representation of the number 24 , to become the 512 -bit padded message.

$$
\underbrace{0110001}_{" \mathrm{a} "} \underbrace{01100010}_{" \mathrm{~b}^{\prime}} \underbrace{01100011}_{" \mathrm{c}^{\prime \prime}} 1 \overbrace{00 \ldots 00}^{423} \overbrace{00 \ldots \underbrace{011000}_{l=24}}^{64}
$$

## BWM384 and BMW512

Suppose that the length of the message $M$ is $l$ bits. Append the bit " 1 " to the end of the message, followed by $k$ zero bits, where $k$ is the smallest, non-negative solution to the equation $l+1+k \equiv$ $960 \bmod 1024$. Then append the 64 -bit block that is equal to the number $l$ expressed using its littleendian binary representation. For example, the ( 8 -bit ASCII) message "abc" has length $8 \times 3=24$, so the message is padded with the bit " 1 ", then $960-(24+1)=935$ zero bits, and then the 64 -bit binary representation of the number 24 , to become the 1024-bit padded message.

$$
\underbrace{01100001}_{" \mathrm{a}^{\prime \prime}} \underbrace{01100010}_{" \mathrm{~b}^{\prime \prime}} \underbrace{01100011}_{" \mathrm{c}^{\prime \prime}} 1 \overbrace{00 \ldots 00}^{935} \overbrace{00 \ldots \underbrace{011000}_{l=24}}^{64}
$$

### 1.5.2 Parsing the message

After a message has been padded, it must be parsed into $N m$-bit blocks before the hash computation can begin.

## BWM224 and BMW256

For BMW224 and BMW256, the padded message is parsed into $N$ 512-bit blocks, $M^{(1)}, M^{(2)}, \ldots$, $M^{(N)}$. Since the 512 bits of the input block may be expressed as sixteen 32-bit words, the first 32 bits of message block $i$ are denoted $M_{0}^{(i)}$, the next 32 bits are $M_{1}^{(i)}$, and so on up to $M_{15}^{(i)}$.

Concretely, for the message $M=$ "abc", the padded and parsed message is represented in Table 1.4. Due to the little-endian nature of Blue Midnight Wish notice the little-endian order of the bytes in $M_{i}$ as well as the "swapped" order between $M_{14}$ and $M_{15}$.

| $M_{0}=0 \times 80636261$ | $M_{1}=0 \times 00000000$ |
| :---: | :---: |
| $M_{2}=0 \times 00000000$ | $M_{3}=0 \times 00000000$ |
| $M_{4}=0 \times 00000000$ | $M_{5}=0 \times 00000000$ |
| $M_{6}=0 \times 00000000$ | $M_{7}=0 \times 00000000$ |
| $M_{8}=0 \times 00000000$ | $M_{9}=0 \times 00000000$ |
| $M_{10}=0 \times 00000000$ | $M_{11}=0 \times 00000000$ |
| $M_{12}=0 \times 00000000$ | $M_{13}=0 \times 00000000$ |
| $M_{14}=0 \times 00000018$ | $M_{15}=0 \times 00000000$ |

Table 1.4: Values for $M$ after the padding of the message "abc" for BMW224/256.

## BWM384 and BMW512

For BMW384 and BMW512, the padded message is parsed into $N$ 1024-bit blocks, $M^{(1)}, M^{(2)}, \ldots$, $M^{(N)}$. Since the 1024 bits of the input block may be expressed as sixteen 64 -bit words, the first 64 bits of message block $i$ are denoted $M_{0}^{(i)}$, the next 64 bits are $M_{1}^{(i)}$, and so on up to $M_{15}^{(i)}$.

Concretely, for the message $M=$ "abc", the padded and parsed message is represented in Table 1.5. Due to the little-endian nature of Blue Midnight Wish notice the little-endian order of the bytes in $M_{i}$.

### 1.5.3 Setting the initial double pipe value $H^{(0)}$

Before hash computation begins for each of the hash algorithms, the initial double pipe value $H^{(0)}$ must be set. The size and the value of words in $H^{(0)}$ depends on the message digest size $n$. As it is shown in the following subsections, the constants consist of concatenation of consecutive natural numbers. Since BMW224 is the same as BMW256 except for the final truncation, they

| $M_{0}=0 \times 0000000080636261$ | $M_{1}=0 \times 0000000000000000$ |
| :---: | :---: |
| $M_{2}=0 \times 0000000000000000$ | $M_{3}=0 \times 0000000000000000$ |
| $M_{4}=0 \times 0000000000000000$ | $M_{5}=0 \times 0000000000000000$ |
| $M_{6}=0 \times 0000000000000000$ | $M_{7}=0 \times 0000000000000000$ |
| $M_{8}=0 \times 0000000000000000$ | $M_{9}=0 \times 0000000000000000$ |
| $M_{10}=0 \times 0000000000000000$ | $M_{11}=0 \times 0000000000000000$ |
| $M_{12}=0 \times 0000000000000000$ | $M_{13}=0 \times 0000000000000000$ |
| $M_{14}=0 \times 0000000000000000$ | $M_{15}=0 \times 0000000000000018$ |

Table 1.5: Values for $M$ after the padding of the message "abc" for BMW384/512
have to have different initial values. Thus, the initial double pipe of BMW224 starts from the byte value $0 \times 00$ and takes all 64 successive byte values up to the value $0 \times 3 F$. Then, the initial double pipe of BMW256 starts from the byte value $0 \times 40$ and takes all 64 successive byte values up to the value 0x7F. The situation is the same with BMW384 and BMW512, but since now the variables are 64-bit long, the initial double pipe of BMW384 starts from the byte value $0 \times 00$ and takes all 128 successive byte values up to the value $0 \times 7 \mathrm{~F}$ and the initial double pipe of BMW512 starts from the byte value $0 \times 80$ and takes all 128 successive byte values up to the value 0 xFF . These constants enable efficient implementation.

## BWM224

For BMW224, the initial double pipe value $H^{(0)}$ shall consist of the sixteen 32-bit words given in Table 1.6.

| $H_{0}^{(0)}=0 \times 00010203$ | $H_{1}^{(0)}=0 \times 04050607$ |
| :--- | :--- |
| $H_{2}^{(0)}=0 \times 08090$ A0B | $H_{3}^{(0)}=0 \times 0$ C0D0E0F |
| $H_{4}^{(0)}=0 \times 10111213$ | $H_{5}^{(0)}=0 \times 14151617$ |
| $H_{6}^{(0)}=0 \times 18191 \mathrm{~A} 1 \mathrm{~B}$ | $H_{7}^{(0)}=0 \times 1 \mathrm{C} 1 \mathrm{D} 1 \mathrm{E} 1 \mathrm{~F}$ |
| $H_{8}^{(0)}=0 \times 20212223$ | $H_{9}^{(0)}=0 \times 24252627$ |
| $H_{10}^{(0)}=0 \times 28292 \mathrm{~A} 2 \mathrm{~B}$ | $H_{11}^{(0)}=0 \times 2 \mathrm{C} 2 \mathrm{D} 2 \mathrm{E} 2 \mathrm{~F}$ |
| $H_{12}^{(0)}=0 \times 30313233$ | $H_{13}^{(0)}=0 \times 34353637$ |
| $H_{14}^{(0)}=0 \times 38393 \mathrm{~A} 3 \mathrm{~B}$ | $H_{15}^{(0)}=0 \times 3 \mathrm{C} 3 \mathrm{D} 3 \mathrm{E} 3 \mathrm{~F}$ |

Table 1.6: Initial double pipe $H^{(0)}$ for BMW224

## BWM256

For BMW256, the initial double pipe value $H^{(0)}$ shall consist of the sixteen 32-bit words given in Table 1.7.

$$
\begin{array}{|l|l|}
\hline H_{0}^{(0)}=0 \times 40414243 & H_{1}^{(0)}=0 \times 44454647 \\
H_{2}^{(0)}=0 \times 48494 \mathrm{~A} 4 \mathrm{~B} & H_{3}^{(0)}=0 \mathrm{x} 4 \mathrm{C} 4 \mathrm{D} 4 \mathrm{E} 4 \mathrm{~F} \\
H_{4}^{(0)}=0 \times 50515253 & H_{5}^{(0)}=0 \times 54555657 \\
H_{6}^{(0)}=0 \times 58595 \mathrm{~A} 5 \mathrm{~B} & H_{7}^{(0)}=0 \times 5 \mathrm{C} 5 \mathrm{D} 5 \mathrm{E} 5 \mathrm{~F} \\
H_{8}^{(0)}=0 \times 60616263 & H_{9}^{(0)}=0 \times 64656667 \\
H_{10}^{(0)}=0 \times 68696 \mathrm{~A} 6 \mathrm{~B} & H_{11}^{(0)}=0 \times 6 \mathrm{C} 6 \mathrm{D} 6 \mathrm{E} 6 \mathrm{~F} \\
H_{12}^{(0)}=0 \times 70717273 & H_{13}^{(0)}=0 \times \mathrm{x} 4757677 \\
H_{14}^{(0)}=0 \times 78797 \mathrm{~A} 7 \mathrm{~B} & H_{15}^{(0)}=0 \mathrm{x} 7 \mathrm{C} 7 \mathrm{D} 7 \mathrm{E} 7 \mathrm{~F} \\
\hline
\end{array}
$$

Table 1.7: Initial double pipe $H^{(0)}$ for BMW256

## BWM384

For BMW384, the initial double pipe value $H^{(0)}$ shall consist of the sixteen 64-bit words given in Table 1.8.

| $H_{0}^{(0)}=0 \times 0001020304050607$ | $H_{1}^{(0)}=0 \times 08090$ AOB0C0D0E0F |
| :--- | :--- |
| $H_{2}^{(0)}=0 \times 1011121314151617$ | $H_{3}^{(0)}=0 \times 18191$ A1B1C1D1E1F |
| $H_{4}^{(0)}=0 \times 2021222324252627$ | $H_{5}^{(0)}=0 \times 28292 A 2 B 2 C 2 D 2 E 2 F$ |
| $H_{6}^{(0)}=0 \times 3031323334353637$ | $H_{7}^{(0)}=0 \times 38393 A 3 B 3 C 3 D 3 E 3 F$ |
| $H_{8}^{(0)}=0 \times 4041424344454647$ | $H_{9}^{(0)}=0 \times 48494 A 4 B 4 C 4 D 4 E 4 F$ |
| $H_{10}^{(0)}=0 \times 5051525354555657$ | $H_{11}^{(0)}=0 \times 58595 A 5 B 5 C 5 D 5 E 5 F$ |
| $H_{12}^{(0)}=0 \times 6061626364656667$ | $H_{13}^{(0)}=0 \times 68696 A 6 B 6 C 6 D 6 E 6 F$ |
| $H_{14}^{(0)}=0 \times 7071727374757677$ | $H_{15}^{(0)}=0 \times 78797 A 7 B 7 C 7 D 7 E 7 F$ |

Table 1.8: Initial double pipe $H^{(0)}$ for BMW384

## BWM512

For BMW512, the initial double pipe value $H^{(0)}$ shall consist of the sixteen 64-bit words given in Table 1.9.

| $H_{0}^{(0)}=0 \times 8081828384858687$ | $H_{1}^{(0)}=0 \times 88898 A 8 B 8 C 8 D 8 E 8 F$ |
| :--- | :--- |
| $H_{2}^{(0)}=0 \times 9091929394959697$ | $H_{3}^{(0)}=0 \times 98999 A 9 B 9 C 9 D 9 E 9 F$ |
| $H_{4}^{(0)}=0 x A 0 A 1 A 2 A 3 A 4 A 5 A 6 A 7$ | $H_{5}^{(0)}=0 \times A 8 A 9 A A A B A C A D A E A F$ |
| $H_{6}^{(0)}=0 x B 0 B 1 B 2 B 3 B 4 B 5 B 6 B 7$ | $H_{7}^{(0)}=0 x B 8 B 9 B A B B B C B D B E B F$ |
| $H_{8}^{(0)}=0 x C 0 C 1 C 2 C 3 C 4 C 5 C 6 C 7$ | $H_{9}^{(0)}=0 x C 8 C 9 C A C B C C C D C E C F$ |
| $H_{10}^{(0)}=0 x D 0 D 1 D 2 D 3 D 4 D 5 D 6 D 7$ | $H_{11}^{(0)}=0 x D 8 D 9 D A D B D C D D D E D F$ |
| $H_{12}^{(0)}=0 x E 0 E 1 E 2 E 3 E 4 E 5 E 6 E 7$ | $H_{13}^{(0)}=0 x E 8 E 9 E A E B E C E D E E E F$ |
| $H_{14}^{(0)}=0 x F 0 F 1 F 2 F 3 F 4 F 5 F 6 F 7$ | $H_{15}^{(0)}=0 \times F 8 F 9 F A F B F C F D F E F F$ |

Table 1.9: Initial double pipe $H^{(0)}$ for BMW512

Chapter 1: Algorithm Specifics

# Description of the Hash Algorithm Blue Midnight Wish 

### 2.1 Generic description for all variants of the Blue Midnight Wish

First we are giving a generic description for all variants of the Blue Midnight Wish hash algorithm. Then, in the following subsections we will give a detailed functional description for the specific variants of the BLUE Midnight Wish hash algorithm for the four different message digest sizes: $n=224, n=256, n=384$ and $n=512$ bits.

In the generic description we are using three functions:

1. The first function is $f_{0}:\{0,1\}^{2 m} \rightarrow\{0,1\}^{m}$. It takes two arguments $M^{(i)}$ and $H^{(i-1)}$ each of $m$ bits and for any value $H^{(i-1)}$ it bijectively transforms $M^{(i)}$. Here, $M^{(i)}$ is the $i$-th message block and $H^{(i-1)}$ is the current value of the double pipe. The result $Q_{a}^{(i)}=f_{0}\left(M^{(i)}, H^{(i-1)}\right)=$ $\mathbf{A}_{2}\left(\mathbf{A}_{1}\left(M^{(i)} \oplus H^{(i-1)}\right)+R O T L^{1}\left(H^{(i-1)}\right)\right.$, is the first part of the extended (quadrupled) pipe. The concrete definition of the bijections $\mathbf{A}_{1}, \mathbf{A}_{2}:\{0,1\}^{m} \rightarrow\{0,1\}^{m}$ will be given later. We denote by $\operatorname{ROTL}^{1}\left(H^{(i-1)}\right)=\left(H_{1}^{(i-1)}, H_{2}^{(i-1)}, \ldots, H_{15}^{(i-1)}, H_{0}^{(i-1)}\right)$ the rotation by one position to the left of the vector $\left(H_{0}^{(i-1)}, H_{1}^{(i-1)}, \ldots, H_{15}^{(i-1)}\right)$ and by $Q_{a}^{(i)}=\left(Q_{0}^{(i)}, \ldots, Q_{15}^{(i)}\right)$.
2. The second function $f_{1}$ takes three arguments: a message block $M^{(i)}$ of $m$ bits, the current value of the double pipe $H^{(i-1)}$ and the value of $Q_{a}^{(i)}$ of $m$ bits, to produce the second part $Q_{b}^{(i)}=\left(Q_{16}^{(i)}, \ldots, Q_{31}^{(i)}\right)$ of the extended (quadrupled) pipe $Q^{(i)}$. The function can be briefly described as $f_{1}:\{0,1\}^{3 m} \rightarrow\{0,1\}^{m}$, and $Q_{b}^{(i)}=f_{1}\left(M^{(i)}, H^{(i-1)}, Q_{a}^{(i)}\right)$. For any given value $H^{(i-1)}$ it is a multipermutation between $M^{(i)}, Q_{a}^{(i)}$ and $Q_{b}^{(i)}$, i.e. for a given pair $\left(M^{(i)}, Q_{a}^{(i)}\right)$ it uniquely computes $Q_{b}^{(i)}$, for a given pair $\left(M^{(i)}, Q_{b}^{(i)}\right)$ it uniquely computes $Q_{a}^{(i)}$ and for a


Figure 2.1: A graphic representation of the Blue Midnight Wish hash algorithm.
given pair $\left(Q_{a}^{(i)}, Q_{b}^{(i)}\right)$ it uniquely computes $M^{(i)}$.
3. For the third function $f_{2}$ we are using the term folding to describe its mapping property to map $3 m$ bits to $m$ bits. It takes two arguments: a message block $M^{(i)}$ of $m$ bits and the current value of the extended pipe $Q^{(i)}=\left(Q_{a}^{(i)}, Q_{b}^{(i)}\right)$ which has $2 m$ bits, to produce a new double pipe $H^{(i)}$ of $m$ bits. So, $f_{2}:\{0,1\}^{3 m} \rightarrow\{0,1\}^{m}$ and $H^{(i)}=f_{2}\left(M^{(i)}, Q^{(i)}\right) \equiv f_{2}\left(M^{(i)}, Q_{a}^{(i)}, Q_{b}^{(i)}\right)$.

The generic description of the Blue Midnight Wish hash algorithm is given in Table 2.1. A graphic representation of the Blue Midnight Wish hash algorithm is given in the Figure 2.1 and its compression function is given in the Figure 2.2.

The function $f_{0}:\{0,1\}^{2 m} \rightarrow\{0,1\}^{m}$ is defined in the Table 2.2.
The function $f_{1}:\{0,1\}^{3 m} \rightarrow\{0,1\}^{m}$ is defined in the Table 2.3.
The function $f_{2}:\{0,1\}^{3 m} \rightarrow\{0,1\}^{m}$ is defined in the Table 2.4.

### 2.1.1 BMW224 and BMW256

BMW224 and BMW256 may be used to hash a message $M$, having a length of $l$ bits, where $0 \leq$ $l<2^{64}$. The algorithms use

1. sixteen 32 -bit working variables that are part of the double pipe, and
2. additional sixteen 32 -bit working variables that together with the variables of the double pipe, make the extended (quadruple) pipe.

## Algorithm: Blue Midnight Wish

Input: Message $M$ of length $/$ bits, and the message digest size $n$. Output: A message digest Hash, that is $n$ bits long.

1. Preprocessing
(a) Pad the message $M$.
(b) Parse the padded message into $N$, $m$-bit message blocks, $M^{(1)}, M^{(2)}, \ldots, M^{(N)}$.
(c) Set the initial value of the double pipe $H^{(0)}$.
2. Hash computation

$$
\begin{aligned}
& \text { For } i=1 \text { to } N \\
& \left\{\begin{array}{l}
Q_{a}^{(i)}=f_{0}\left(M^{(i)}, H^{(i-1)}\right) ; \\
Q_{b}^{(i)}=f_{1}\left(M^{(i)}, H^{(i-1)}, Q_{a}^{(i)}\right) ; \\
H^{(i)}=f_{2}\left(M^{(i)}, Q_{a}^{(i)}, Q_{b}^{(i)}\right) ;
\end{array}\right.
\end{aligned}
$$

3. Finalization

$$
\begin{aligned}
& Q_{a}^{\text {final }}=f_{0}\left(H^{(N)}, \text { CONST }^{\text {final }}\right) ; \\
& Q_{b}^{\text {final }}=f_{1}\left(H^{(N)}, \text { CONST }^{\text {final }}, Q_{a}^{\text {final }}\right) ; \\
& H^{\text {final }}=f_{2}\left(H^{(N)}, Q_{a}^{\text {final }}, Q_{b}^{\text {final }}\right) ;
\end{aligned}
$$

4. $H a s h=T a k e \_n \_$Least_Significant_Bits $\left(H^{f i n a l}\right)$.

Table 2.1: A generic description of the Blue Midnight Wish hash algorithm


Figure 2.2: Graphical representation of the compression function in BLUE MIDNIGHT WISH

The words of the quadruple pipe are labeled $Q_{0}^{(i)}, Q_{1}^{(i)}, \ldots, Q_{31}^{(i)}$. The words of the initial double pipe are labeled $H_{0}^{(i-1)}, H_{1}^{(i-1)}, \ldots, H_{15}^{(i-1)}$, which will hold the initial double pipe value $H^{(0)}$, replaced by each successive intermediate double pipe value (after each message block is processed), $H^{(i)}$, and ending with the final double pipe value $H^{(N)}$. BMW224 and BMW256 also use two temporary 32 -bit words $X L$ and $X H$. The final result of BMW224 is a 224-bit message digest that are the least significant 224 bits from the final hash value $H^{\text {final }}$ i.e. the values $\left(H_{9}^{(f i n a l)}, \ldots, H_{15}^{(f i n a l)}\right)$, and the final result of BMW256 is a 256-bit message digest that are the least significant 256 bits from the final hash value $H^{f i n a l}$ i.e. the values $\left(H_{8}^{(f i n a l)}, \ldots, H_{15}^{(f i n a l)}\right)$.

2. Further bijective transform of $W_{j}^{(i)}, j=0, \ldots, 15$ :

$$
\begin{array}{llll}
Q_{0}^{(i)}=s_{0}\left(W_{0}^{(i)}\right)+H_{1}^{(i-1)} ; & Q_{1}^{(i)}=s_{1}\left(W_{1}^{(i)}\right)+H_{2}^{(i-1)} ; & Q_{2}^{(i)}=s_{2}\left(W_{2}^{(i)}\right)+H_{3}^{(i-1)} ; & Q_{3}^{(i)}=s_{3}\left(W_{3}^{(i)}\right)+H_{4}^{(i-1)} ; \\
Q_{4}^{(i)}=s_{4}\left(W_{4}^{(i)}\right)+H_{5}^{(i-1)} ; & Q_{5}^{(i)}=s_{0}\left(W_{5}^{(i)}\right)+H_{6}^{(i-1)} ; & Q_{6}^{(i)}=s_{1}\left(W_{6}^{(i)}\right)+H_{7}^{(i-1)} ; & Q_{7}^{(i)}=s_{2}\left(W_{7}^{(i)}\right)+H_{8}^{(i-1)} ; \\
Q_{8}^{(i)}=s_{3}\left(W_{8}^{(i)}\right)+H_{9}^{(i-1)} ; & Q_{9}^{(i)}=s_{4}\left(W_{9}^{(i)}\right)+H_{10}^{(i-1) ;} ; & Q_{10}^{(i)}=s_{0}\left(W_{10}^{(i)}\right)+H_{11}^{(i-1)} ; & Q_{11}^{(i)}=s_{1}\left(W_{11}^{(i)}\right)+H_{12}^{(i-1)} ; \\
Q_{12}^{(i)}=s_{2}\left(W_{12}^{(i)}\right)+H_{13}^{(i-1)} ; & Q_{13}^{(i)}=s_{3}\left(W_{13}^{(i)}\right)+H_{14}^{(i-1) ;} ; & Q_{14}^{(i)}=s_{4}\left(W_{14}^{(i)}\right)+H_{15}^{(i-1)} ; & Q_{15}^{(i)}=s_{0}\left(W_{15}^{(i)}\right)+H_{0}^{(i-1)} ;
\end{array}
$$

Table 2.2: Definition of the function $f_{0}$ of BLuE Midnight Wish

| $f_{1}:\{0,1\}^{3 m} \rightarrow\{0,1\}^{m}$ |
| :--- |
| Input: Message block $M^{(i)}=\left(M_{0}^{(i)}, M_{1}^{(i)}, \ldots, M_{15}^{(i)}\right)$, the previous double pipe $H^{(i-1)}=\left(H_{0}^{(i-1)}, H_{1}^{(i-1)}, \ldots, H_{15}^{(i-1)}\right)$ and |
| the first part of the quadruple pipe $Q_{a}^{(i)}=\left(Q_{0}^{(i)}, Q_{1}^{(i)}, \ldots, Q_{15}^{(i)}\right)$. |
| Output: Second part of the quadruple pipe $Q_{b}^{(i)}=\left(Q_{16}^{(i)}, Q_{17}^{(i)}, \ldots, Q_{31}^{(i)}\right)$. |
| 1. Double pipe expansion according to the tunable parameters ExpandRounds ${ }_{1}$ and ExpandRounds ${ }_{2}$. |
| 1.1 For $i i=0$ to ExpandRounds $1-1$ |
| $Q_{i i+16}^{(i)}=$ expand $_{1}(i i+16)$ |
| 1.2 For $i i=$ ExpandRounds $_{1}$ to ExpandRounds $s_{1}+$ ExpandRounds $_{2}-1$ |
| $Q_{i i+16}^{(i)}=$ expand $_{2}(i i+16)$ |

Table 2.3: Definition of the function $f_{1}$ of BLUE Midnight Wish


Table 2.4: Definition of the folding function $f_{2}$ of BLUE Midnight Wish

### 2.1.2 BMW384 and BMW512

BMW384 and BMW512 may be used to hash a message $M$, having a length of $l$ bits, where $0 \leq$ $l<2^{64}$. The algorithms use

1. sixteen 64-bit working variables that are part of the double pipe, and
2. additional sixteen 64 -bit working variables that together with the variables of the double pipe, make the extended (quadrupled) pipe.

The words of the quadruple pipe are labeled $Q_{0}^{(i)}, Q_{1}^{(i)}, \ldots, Q_{31}^{(i)}$. The words of the initial double pipe are labeled $H_{0}^{(i)}, H_{1}^{(i)}, \ldots, H_{15}^{(i)}$, which will hold the initial double pipe value $H^{(0)}$, replaced by each successive intermediate double pipe value (after each message block is processed), $H^{(i)}$, and ending with the final double pipe value $H^{(N)}$. BMW384 and BMW512 also use two temporary 64-bit words XL and XH. The final result of BMW384 is a 384-bit message digest that are the least significant 384 bits from the final hash value $H^{\text {final }}$ i.e. the values $\left(H_{10}^{(f i n a l)}, \ldots, H_{15}^{(f i n a l)}\right)$, and the final result of BMW512 is a 512-bit message digest that are the least significant 512 bits from the final hash value $H^{f \text { inal }}$ i.e. the values $\left(H_{8}^{(f \text { final })}, \ldots, H_{15}^{(f i n a l)}\right)$.

## Design Rationale

### 3.1 Reasons for default little-endian design

Some of the earlier versions of BLUE Midnight Wish were designed to be big-endian by default. However, as the designing phase was coming to its end, and we started the optimization phase, we changed the default design to be little-endian since an overwhelming majority of CPU platforms in the world are little-endian.

### 3.2 Reasons for using double pipe iterative structure

In the design of BLUE Midnight Wish we have decided to incorporate the suggestions of Lucks $[2,3]$ and Coron et al. [4] by setting the size of the chaining pipe to be twice the hash digest size. This design avoids the weaknesses against the generic attacks of Joux [5] and Kelsy and Schneier [6], thereby guaranteeing resistance against a generic multicollision attack and length extension attacks.

Additionally, as we will see later, using the double pipe concept in combination with the used nonlinear bijections is an effective precaution against differential attacks, because the attacker will have to use twice the number of variables in the differential paths than in a single pipe.

### 3.3 Rationale for constants used in Blue Midnight Wish

### 3.3.1 Constants in logical functions

The logical functions $s_{0}, s_{1}, s_{2}$ and $s_{3}$ are chosen in such a way that they satisfy the following criteria:

- They are bijections in $\{0,1\}^{32} \rightarrow\{0,1\}^{32}$ (resp. in $\{0,1\}^{64} \rightarrow\{0,1\}^{64}$ ).
- They have different pairs of 1-bit, 2-bits or 3-bits shifts to the left and to the right.
- They have different pairs of rotations to the left, in such a way that one rotation is less than $w / 2, w=32,64$, and the other rotation is bigger than $w / 2$.
- The values of the rotations that are less than $w / 2$ are in the interval of $\pm 2$ (resp. $\pm 4$ ) around numbers $\{2,6,10,14\}$ (resp. $\{4,12,20,28\}$ ).
- The values of the rotations that are bigger than $w / 2$ are in the interval of $\pm 2$ (resp. $\pm 4$ ) around numbers $\{18,22,26,30\}$ (resp. $\{36,42,50,58\}$ ).

By computer search we have found hundreds of such bijections and from them we have chosen the four particular functions $s_{0}, s_{1}, s_{2}$ and $s_{3}$. The role of these logical functions is to diffuse a one-bit difference into 3 or 4 bits differences.

The logical functions $s_{4}$ and $s_{5}$ are bijections in $\{0,1\}^{32} \rightarrow\{0,1\}^{32}$ (resp. in $\{0,1\}^{64} \rightarrow\{0,1\}^{64}$ ). They have only one shift to the right for just one or two bits. Their role is to spread a one-bit differences mostly into two bits (if the difference bit is the right-most or the bit next to the rightmost bit, then these functions keep a one-bit difference as a one-bit difference).
Logical functions $r_{1}, \ldots, r_{7}$ are rotations with the values that were chosen uniformly at random in the interval $[1, w-1]$.

### 3.3.2 Constants in the expansion part

In the expansion function $f_{1}$ we use the constants $K_{j}=j \times(0 \times 05555555), j=16,17, \ldots, 31$ for BMW224 and BMW256, or the constants $K_{j}=j \times(0 \times 0555555555555555), j=16,17, \ldots, 31$ for BMW384 and BMW512.

The primary reason why we use constants is that we want to avoid the situation that the message $M=(0,0, \ldots, 0) \equiv \mathbf{0}$ and the double pipe value $H=(0,0, \ldots, 0) \equiv \mathbf{0}$ are a fixed point. Let
us for a moment omit the upper index ${ }^{(i)}$ in our notations. If we have in mind that ( $Q_{a}, Q_{b}$ ) = $\left(f_{0}(M, H), f_{1}\left(M, H, f_{0}(M, H)\right)\right)$, then if $f_{1}$ does not have constants we will have the situation that

$$
(\mathbf{0}, \mathbf{0})=\left(f_{0}(\mathbf{0}, \mathbf{0}), f_{1}\left(\mathbf{0}, \mathbf{0}, f_{0}(\mathbf{0}, \mathbf{0})\right)\right) .
$$

We have chosen $0 \times 05555555$ and $0 \times 0555555555555555$ as a basis for obtaining 16 constants in the expansion function because we find that their binary representation as a sequence of alternating 0 s and 1 s is good source of variety.

The reason why we choose $0 \times 05555555$ instead of $0 \times 55555555$ is simply to avoid complaints (warnings) of some $C$ compilers that are finding that $16 \times(0 \times 55555555)$ is a constant that goes out of the range of a 32-bit word (the reason is similar for $0 \times 0555555555555555$ ).

### 3.3.3 Constants in the finalization part

In the final invocation of the compression function we have changed the role of the chaining double pipe and the message. Since there is no more message to be digested, the role that the message data was performing in the previous invocations of the compression function is now given to the last obtained double pipe $H^{(N)}$. In such a case the role of the chaining double pipe is fixed to a constant that we denote as: CONST ${ }^{\text {final }}$.

We have chosen 16 components of the vector CONST ${ }^{\text {final }}=\left(\operatorname{CONST}_{0}^{f i n a l}, \ldots, \operatorname{CONST}_{15}^{f i n a l}\right)$ to be

- $\operatorname{CONST}_{j}^{\text {final }}=0$ xaaaaaaa0 $+j, j=0,1, \ldots, 15$ for BMW224 and BMW256.
- $\operatorname{CONST}_{j}^{\text {final }}=0 \times \mathrm{xaaa} a a_{a} a a_{a} a \mathrm{a} a \mathrm{a} 0+j, j=0,1, \ldots, 15$ for BMW384 and BMW512.

By fixing the CONST $T^{\text {final }}$ we are removing one degree of freedom to the attackers who try to find pseudo collisions and pseudo-preimages. Additionally, the final invocation of the compression function is a measure for any attack whereby an attacker can find near collisions or near-pseudocollisions of the compression function of BLUE Midnight Wish.

### 3.4 Rationale for the bijective "Step $\mathbf{1}^{\prime \prime}$ in the function $f_{0}$

Step 1 in the definition of the function $f_{0}$ is a bijective one when either $H^{(i-1)}$ or $M^{(i)}$ are kept constant (or can be seen as a bijective transformation of $M^{(i)} \oplus H^{(i-1)}$ ). For this description we can denote the result of that transformation with an intermediate working variable $W$ :

$$
W=\mathbf{A}_{1} \cdot\left(M^{(i)} \oplus H^{(i-1)}\right),
$$

where we denote $W^{(i)}=\left(W_{0}^{(i)}, W_{1}^{(i)}, \ldots, W_{15}^{(i)}\right)$ and the matrix $\mathbf{A}_{1}$ is a $16 \times 16$ nonsingular matrix in $\mathbb{Z}_{2^{32}}$ and in $\mathbb{Z}_{2^{64}}$. The value of $\mathbf{A}_{1}$ is

$$
\mathbf{A}_{1}=\left(\begin{array}{rrrrrrrrrrrrrrrr}
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & -1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 1 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & -1 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 \\
-1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\
1 & 0 & 0 & -1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
-1 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 1 & 0 & 0
\end{array}\right) .
$$

The matrix $\mathbf{A}_{1}$ was obtained from the matrix

$$
\mathbf{A}^{\prime}{ }_{1}=\left(\begin{array}{llllllllllllllll}
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0
\end{array}\right)
$$

by randomly turning some of the values ' 1 ' into ' -1 '. Note that the product $\mathbf{A}^{\prime}{ }_{1} \cdot M^{(i)}$ can be expressed as:

$$
\mathbf{A}_{1}^{\prime}=\operatorname{ROTR}^{2}\left(M^{(i)}\right)+\operatorname{ROTR}^{3}\left(M^{(i)}\right)+\operatorname{ROTR}^{6}\left(M^{(i)}\right)+\operatorname{ROTR}^{9}\left(M^{(i)}\right)+\operatorname{ROTR}^{11}\left(M^{(i)}\right)
$$

where the operations $\operatorname{ROTR}^{j}\left(M^{(i)}\right)$ are rotations to the right of the vector $M^{(i)}=\left(M_{0}^{(i)}, M_{1}^{(i)}, \ldots, M_{15}^{(i)}\right)$ by $j$ words and " + " means componentwise addition in $\mathbb{Z}_{2^{32}}$ (resp. in $\mathbb{Z}_{2^{64}}$ ). In other words we have that:

$$
\begin{aligned}
\operatorname{ROTR}^{2}\left(M^{(i)}\right) & =\left(M_{14}^{(i)}, M_{15}^{(i)}, M_{0}^{(i)}, \ldots, M_{13}^{(i)}\right) \\
\operatorname{ROTR}^{3}\left(M^{(i)}\right) & =\left(M_{13}^{(i)}, M_{14}^{(i)}, M_{15}^{(i)}, \ldots, M_{12}^{(i)}\right) \\
\operatorname{ROTR}^{6}\left(M^{(i)}\right) & =\left(M_{10}^{(i)}, M_{11}^{(i)}, M_{12}^{(i)}, \ldots, M_{9}^{(i)}\right), \\
\operatorname{ROTR}^{9}\left(M^{(i)}\right) & =\left(M_{7}^{(i)}, M_{8}^{(i)}, M_{9}^{(i)}, \ldots, M_{6}^{(i)}\right) \\
\operatorname{ROTR}^{11}\left(M^{(i)}\right) & =\left(M_{5}^{(i)}, M_{6}^{(i)}, M_{7}^{(i)}, \ldots, M_{4}^{(i)}\right)
\end{aligned}
$$

and

$$
\mathbf{A}_{1}^{\prime} \cdot M^{(i)}=\left(M_{14}^{(i)}+M_{13}^{(i)}+M_{10}^{(i)}+M_{7}^{(i)}+M_{5}^{(i)}, \ldots, M_{13}^{(i)}+M_{12}^{(i)}+M_{9}^{(i)}+M_{6}^{(i)}+M_{4}^{(i)}\right)
$$

It is straightforward to prove the following
Lemma 1. The transformation $\mathbf{A}_{1}^{\prime} \cdot M^{(i)}$ diffuses every one bit difference in the vector $M^{(i)}$ into at least five bits difference.

The matrix $\mathbf{A}_{1}$ is obtained from the matrix $\mathbf{A}_{1}^{\prime}$ by randomly selecting some of the values " 1 " and turning them into " -1 ". It is straightforward to prove the following

Lemma 2. The transformation $\mathbf{A}_{1} \cdot M^{(i)}$ diffuses every one bit difference in the vector $M^{(i)}$ into at least five bits difference.

The reason why we decided to use the transformation $\mathbf{A}_{1} \cdot M^{(i)}$ instead of the transformation $\mathbf{A}^{\prime} \cdot M^{(i)}$ is the fact that in any CPU, the computational costs of addition and subtraction are the same, but the component with mixed usage of additions and subtractions is more complex. It is reasonable to expect that increased complexity also increases the ability to resist cryptanalysis.

### 3.5 Rationale for the bijective "Step 2" in the function $f_{0}$

Step 2 in the definition of the function $f_{0}$ is also a bijective one, but now the bijective transformation is achieved for every component of the vector $W^{(i)}$ by applying transformations $s_{0}, s_{1}, s_{2}, s_{3}$ and $s_{4}$ (see the Table 1.3).

It is easy to prove the following
Lemma 3. The transformations $s_{i}, i=0, \ldots, 5$ and $r_{i}, i=1, \ldots, 7$ defined in the Table 1.3 are bijective transformations in $\{0,1\}^{32}$ (resp. in $\{0,1\}^{64}$ ).

For our analysis of the hash function we denote this bijective Step 2 transformation as $\mathbf{A}_{\mathbf{2}}$ : $\{0,1\}^{16 w} \rightarrow\{0,1\}^{16 w}$. From the composition of Step 1 and Step 2 in the function $f_{0}$ it is clear that

$$
f_{0}\left(M_{i}, H_{i-1}\right) \equiv \mathbf{A}_{2}\left(\mathbf{A}_{1} \cdot\left(M_{i} \oplus H_{i-1}\right)\right)+\operatorname{ROTL}^{1}\left(H_{i-1}\right) .
$$

We denote by $\operatorname{ROTL}^{1}\left(H_{i-1}\right)=\left(H_{1}^{(i-1)}, H_{2}^{(i-1)}, \ldots, H_{15}^{(i-1)}, H_{0}^{(i-1)}\right)$ the rotation by one position to the left of the vector $\left(H_{0}^{(i-1)}, H_{1}^{(i-1)}, \ldots, H_{15}^{(i-1)}\right)$. The reason why we put this additional term $\operatorname{ROTL}^{1}\left(H_{i-1}\right)$ (it was not present in the Round 1 version of Blue Midnight Wish ) is that we installed two actions of a decoupling between $M_{i}$ and $H_{i-1}$ in order to prevent pseudo-attacks that can use the fact that $M_{i} \oplus H_{i-1}=0$ iff $M_{i}=H_{i-1}$. This is the first such decoupling, and the second one is installed in the expansion function $f_{1}()$.

The differential (diffusion) property for $s_{i}, i=0, \ldots, 3$ transformations is summarized in the following

Lemma 4. The transformations $s_{0}, s_{1}, s_{2}$ and $s_{3}$ defined in the Table 1.3 diffuse every one bit difference in their arguments (32-bit or 64-bit words) into 3 or 4 bits of difference.

The differential (diffusion) property for $s_{4}$ and $s_{5}$ transformations is summarized in the following
Lemma 5. The transformations $s_{4}$ and $s_{5}$ defined in the Table 1.3 diffuse every one bit difference in their arguments (32-bit or 64-bit words) into 1 or 2 bits of difference.

The differential (diffusion) property of consecutive application of Step 1 and Step 2 in the function $f_{0}$ can be determined by combining Lemma 4 and Lemma 5 and is summarized in the following

Lemma 6. Every one bit difference in the vector $W^{(i)}$ after Step 1 and Step 2 of the function $f_{0}$ diffuses into 5 words of the the vector $Q_{a}$, and the differences in those 5 words are minimum 1 or 2 bits difference, or minimum 3 or 4 bits difference.

Proof. We have tested all possible one-bit differences with all possible multiple runs of consecutive bit differences that can be obtained with the operations of addition or subtraction modulo $2^{32}$ or modulo $2^{64}$ after Step 1 of the function $f_{0}$. Then we have processed those differences further by $s_{0}, \ldots, s_{3}$, or by $s_{4}$ and $s_{5}$. For the cases when those differences are processed by $s_{0}, \ldots, s_{3}$ we have that the minimum is either 3 or 4 bits, and when we process those differences by $s_{4}$ and $s_{5}$ we have that the minimum is 1 or 2 bits.

### 3.6 Tunable parameters ExpandRounds ${ }_{1}$ and ExpandRounds ${ }_{2}$

The function $f_{1}$ is designed as a weak block cipher as it is described in Section 2.1. It takes arguments $M^{(i)}, H^{(i-1)}$ and $Q_{a}=\left(Q_{0}^{(i)}, Q_{1}^{(i)}, \ldots, Q_{15}^{(i)}\right)$ and computes the values $Q_{b}=\left(Q_{16}^{(i)}, Q_{17}^{(i)}, \ldots, Q_{31}^{(i)}\right)$. Actually for any given value $H^{(i-1)}$, the function $f_{1}$ is a multipermutation between $M^{(i)}, Q_{a}^{(i)}$ and $Q_{b}^{(i)}$. That means that for a given pair $\left(M^{(i)}, Q_{a}^{(i)}\right)$ it uniquely computes $Q_{b}^{(i)}$, for a given pair $\left(M^{(i)}, Q_{b}^{(i)}\right)$ it uniquely computes $Q_{a}^{(i)}$ and for a given pair $\left(Q_{a}^{(i)}, Q_{b}^{(i)}\right)$ it uniquely computes $M^{(i)}$. We are achieving that in 16 expansion steps using two types of expansion functions. The first expansion function $\operatorname{expand}_{1}()$ is used in the beginning of the expansion process. In that function, a difference of a one bit in $M^{(i)}$ or in $Q_{a}$ diffuses much faster than in the second expansion function
expand $_{2}()$. The number of times we will call the first and the second function are connected with the following relation:

$$
\text { ExpandRounds }_{1}+\text { ExpandRounds } s_{2}=16
$$

The function

$$
\begin{array}{rlll}
\operatorname{expand}_{1}(j) & = & s_{1}\left(Q_{j-16}^{(i)}\right) & \\
& +s_{2}\left(Q_{j-15}^{(i)}\right)+s_{3}\left(Q_{j-14}^{(i)}\right)+s_{0}\left(Q_{j-13}^{(i)}\right) \\
& s_{1}\left(Q_{j-12}^{(i)}\right) & & +s_{2}\left(Q_{j-11}^{(i)}\right)+s_{3}\left(Q_{j-10}^{(i)}\right)+s_{0}\left(Q_{j-9}^{(i)}\right) \\
& s_{1}\left(Q_{j-8}^{(i)}\right) & & +s_{2}\left(Q_{j-7}^{(i)}\right)+s_{3}\left(Q_{j-6}^{(i)}\right)+s_{0}\left(Q_{j-5}^{(i)}\right) \\
& + & s_{1}\left(Q_{j-4}^{(i)}\right) & \\
& +s_{2}\left(Q_{j-3}^{(i)}\right)+s_{3}\left(Q_{j-2}^{(i)}\right)+s_{0}\left(Q_{j-1}^{(i)}\right) \\
& + \text { AddElement }(j-16),
\end{array}
$$

is a more complex and more computationally expensive function in the expansion part. However, as a sort of security/performance tradeoff for the computation of the expanded values, we are using the second simplified expand function:

$$
\begin{array}{rlrl}
\operatorname{expand}_{2}(j) & = & Q_{j-16}^{(i)} & \\
& +r_{1}\left(Q_{j-15}^{(i)}\right)+Q_{j-14}^{(i)}+r_{2}\left(Q_{j-13}^{(i)}\right) \\
& +\quad Q_{j-12}^{(i)} & & +r_{3}\left(Q_{j-11}^{(i)}\right)+Q_{j-10}^{(i)}+r_{4}\left(Q_{j-9}^{(i)}\right) \\
& Q_{j-8}^{(i)} & & +r_{5}\left(Q_{j-7}^{(i)}\right)+Q_{j-6}^{(i)}+r_{6}\left(Q_{j-5}^{(i)}\right) \\
& Q_{j-4}^{(i)} & & +r_{7}\left(Q_{j-3}^{(i)}\right)+s_{4}\left(Q_{j-2}^{(i)}\right)+s_{5}\left(Q_{j-1}^{(i)}\right) \\
& + \text { AddElement }(j-16) .
\end{array}
$$

Our recommendation for these tunable parameters is: ExpandRounds $s_{1}=2$, ExpandRounds $_{2}=14$. Here, the term AddElement $(j)$ is computed by the expression

$$
\text { AddElement }(j)=\left(\operatorname{ROTL}^{(j+1)}\left(M_{j}^{(i)}\right)+\operatorname{ROTL}^{(j+4)}\left(M_{j+3}^{(i)}\right)-\operatorname{ROTL}^{(j+11)}\left(M_{j+10}^{(i)}\right)+K_{j+16}\right) \oplus H_{j+7}^{(i)} .
$$

Note that in the Round 1, Blue Midnight Wish had a different AddElement(j) element. Namely, the old expression for this term was:

$$
\text { Old_AddElement }(j)=M_{j}^{(i)}+M_{j+3}^{(i)}-M_{j+10}^{(i)}+K_{j+16} .
$$

However, the old term was giving a chance for an attacker to make changes in the most significant bits of the message and due to the operations of addition, those changes were canceling each other up to the last variable $Q_{31}$, thus giving free-start near collisions in the compression function. The new (tweaked) expression for AddElement $(j)$ rotates the values of the message $M^{(i)}$, and additionally operates with the vector $\operatorname{ROTL}^{7}\left(H_{i-1}\right)=\left(H_{7}^{(i-1)}, H_{8}^{(i-1)}, \ldots, H_{5}^{(i-1)}, H_{6}^{(i-1)}\right)$ which denotes the rotation by seven position to the left of the vector $\left(H_{0}^{(i-1)}, H_{1}^{(i-1)}, \ldots, H_{15}^{(i-1)}\right)$. This is
our second introduction of expressions that decouples the input values of the message $M^{(i)}$ and the chaining double pipe $H^{(i-1)}$ with the particular values from $M^{(i)}$ and $H^{(i-1)}$ that are repeatedly used in the Blue Midnight Wish expressions.

If we take all elements $\operatorname{AddElement}(j)$ for $j=0,1, \ldots, 15$, then we can write them shortly and symbolically as

$$
\text { AddElement }\left(M^{(i)}, H^{(i)}\right)=\left(\mathbf{B}\left(\operatorname{rot} M^{(i)}\right)+K\right) \oplus \operatorname{ROTL} L^{7}\left(H_{i-1}\right) .
$$

The matrix $\mathbf{B}$ is:

$$
\mathbf{B}=\left(\begin{array}{rrrrrrrrrrrrrrrr}
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
-1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right),
$$

and it is nonsingular over $\mathbb{Z}_{2^{w}}$. The vector $\operatorname{rot} M^{(i)}$ is the vector obtained from $M^{(i)}$ where its $j$-th component $(j=0,1, \ldots, 15)$ is rotated to the left for $j+1$ positions i.e.:

$$
\operatorname{rot} M^{(i)}=\left(\operatorname{ROTL}^{1}\left(M_{0}^{(i)}\right), \operatorname{ROTL}^{2}\left(M_{1}^{(i)}\right), \ldots, \operatorname{ROTL}^{15}\left(M_{14}^{(i)}\right), \operatorname{ROTL}^{16}\left(M_{15}^{(i)}\right)\right),
$$

and the constants in the vector $K$ are described in the Section 1.2.

### 3.6.1 Statements, relating to the NIST requirements 2.B.1.

Here we give statements, in relation to the NIST requirements 2.B.1.
I.

The following statements are the same for each digest size $n=224,256,384,512$.
II.

Using two consecutive $\operatorname{expand}_{1}()$ rounds at the beginning of the weak block cipher $f_{1}$ means that the variables $Q_{a}=\left(Q_{0}, \ldots, Q_{15}\right)$ enter the 16 -round block cipher $f_{1}$ in two different linear
combinations of their bits consecutively (excluding $Q_{0}$, which enters the cipher $f_{1}$ directly only once as $s_{1}\left(Q_{0}\right)$ and indirectly in $\left.Q_{17}, \ldots, Q_{31}\right)$. For instance $Q_{1}$ enters $f_{1}$ in the first two rounds directly as $s_{2}\left(Q_{1}\right)$ and $s_{1}\left(Q_{1}\right), Q_{2}$ enters $f_{1}$ in the first two rounds directly as $s_{3}\left(Q_{2}\right)$ and $s_{2}\left(Q_{2}\right)$, etc. The more rounds of $\operatorname{expand}_{1}()$ are used, the more linear combinations of variables of $Q_{a}$ enter the cipher $f_{1}$.

## III.

By using more rounds of expand $_{1}()$ we can increase the strength (and the complexity) of the cipher $f_{1}$, and thus the security of BLUE Midnight Wish, but we will decrease the speed.
IV.

By using two different round functions $\operatorname{expand}_{1}()$ and $\operatorname{expand}_{2}()$ we increase the difficulty of finding overall differential paths, because the differentials for the first function $\operatorname{expand}_{1}()$ and for the second function expand $_{2}()$ are completely different.

## V.

We are not aware of any weaknesses even for ExpandRounds ${ }_{1}=0$ and ExpandRounds ${ }_{2}=16$ or ExpandRounds $s_{1}=16$ and ExpandRounds $s_{2}=0$ or any other combination for ExpandRounds ${ }_{1}+$ ExpandRounds $s_{2}=16$, but we propose ExpandRounds ${ }_{1}=2$ as an optimal tradeoff between security and efficiency.

### 3.7 Cryptanalysis of Blue Midnight Wish

### 3.7.1 Bijective parts in the compression function of Blue Midnight Wish

Here we will write the compression function in such a way that we will emphasize all its functional entities. Later on, this representation will help us to perform a cryptanalysis of the compression function.

First let us adopt the following notation for this and the next section:

1. Sometimes we omit the upper index ${ }^{(i)}$ in our notations.
2. In that case we denote the $i$-th message block as $M_{i}$ (instead of $M^{(i)}$ ).
3. Also, in that case we denote the $(i-1)$-th double pipe as $H_{i-1}$ (instead of $H^{(i-1)}$ ).
4. Also in that case we denote the final output from the function $f_{2}$ as $H_{i}$ i.e. $H_{i}=f_{2}\left(M_{i}, Q_{a}, Q_{b}\right)$ (instead of $H^{(i)}$ ).

Having in mind the definition of the function $f_{2}$ given in Table 2.4 we can rewrite the function $f_{2}$ as follows.

Let $f_{3}:\{0,1\}^{2 m} \rightarrow\{0,1\}^{m}$ be defined as:

Further on, let $f_{4}:\{0,1\}^{2 m} \rightarrow\{0,1\}^{m}$ be defined as:

$$
f_{4}\left(Q_{a}, Q_{b}\right)=\left(\begin{array}{rcc}
X L \oplus & Q_{24}^{(i)} & \oplus Q_{0}^{(i)} \\
X L \oplus & Q_{25}^{(i)} & \oplus Q_{1}^{(i)} \\
X L \oplus & Q_{26}^{(i)} & \oplus Q_{2}^{(i)} \\
X L \oplus & Q_{2( }^{(i)} & \oplus Q_{3}^{(i)} \\
X L \oplus & Q_{28}^{(i)} & \oplus Q_{4}^{(i)} \\
X L \oplus & Q_{29}^{(i)} & \oplus Q_{5}^{(i)} \\
X L \oplus & Q_{30}^{(i)} & \oplus Q_{6}^{(i)} \\
X L \oplus & Q_{31}^{(i)} & \oplus Q_{7}^{(i)} \\
S H L^{8}(X L) \oplus & Q_{23}^{(i)} & \oplus Q_{8}^{(i)} \\
S H R^{6}(X L) \oplus & Q_{16}^{(i)} & \oplus Q_{9}^{(i)} \\
S H L^{6}(X L) \oplus & Q_{17}^{(i)} & \oplus Q_{10}^{(i)} \\
S H L^{4}(X L) \oplus & Q_{18}^{(i)} & \oplus Q_{11}^{(i)} \\
S H R^{3}(X L) \oplus & Q_{19}^{(i)} & \oplus Q_{12}^{(i)} \\
S H R^{4}(X L) \oplus & Q_{20}^{(i)} & \oplus Q_{13}^{(i)} \\
S H R^{7}(X L) \oplus & Q_{21}^{(i)} & \oplus Q_{14}^{(i)} \\
S H R^{2}(X L) \oplus & Q_{22}^{(i)} & \oplus Q_{15}^{(i)}
\end{array}\right)
$$

Finally for any $X=\left(X_{0}, X_{1}, \ldots, X_{15}\right)$ where $X_{i}$ are $w$-bit words $(w=32,64)$, let us define the
function $f_{5}:\{0,1\}^{16 w} \rightarrow\{0,1\}^{16 w}$ as:

$$
f_{5}(X)=\left(\begin{array}{r}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\operatorname{ROTL}^{9}\left(X_{4}\right) \\
\operatorname{ROTL}^{10}\left(X_{5}\right) \\
\operatorname{ROTL}^{11}\left(X_{6}\right) \\
\operatorname{ROTL}^{12}\left(X_{7}\right) \\
\operatorname{ROTL}^{13}\left(X_{0}\right) \\
\operatorname{ROTL}^{14}\left(X_{1}\right) \\
\operatorname{ROTL}^{15}\left(X_{2}\right) \\
\operatorname{ROTL}^{16}\left(X_{3}\right)
\end{array}\right)
$$

Now the final output from the $f_{2}$ function is $H_{i}=\left(H_{0}, H_{1}, \ldots, H_{15}\right)$ and can be rewritten as:

$$
\begin{equation*}
H_{i}=f_{2}\left(M_{i}, Q_{a}, Q_{b}\right) \equiv f_{3}\left(M_{i}, Q_{b}\right)+f_{4}\left(Q_{a}, Q_{b}\right)+f_{5}\left(f_{3}\left(M_{i}, Q_{b}\right)+f_{4}\left(Q_{a}, Q_{b}\right)\right) \tag{3.7.1}
\end{equation*}
$$

One of the basic security properties of BLUE MIDNIGHT WISH is its nonlinear folding function $f_{2}$. We describe here one specially designed part of this function.

Let us denote by $L_{a}$ the the following function:

Further on, let us denote by $L_{b}$ the function:

Finally, let us define the transformation $L:\{0,1\}^{16 w} \rightarrow\{0,1\}^{16 w}$ as $L \equiv L_{a} \oplus L_{b}$ i.e.:

Theorem 1. The transformation $L:\{0,1\}^{16 w} \rightarrow\{0,1\}^{16 w}$ is a bijection for both values $w=32$ and $w=64$.

Proof. A direct linear algebra check of the determinant of the corresponding matrix for the transformation $L$ for both cases $w=32$ and $w=64$ shows that the determinant is 1 (in $G F(2)$ ).

The constants for shifting left or right used in the transformation $L$ were found by a computer search, such that L is bijective transformation both for $w=32$ and $w=64$.

The following theorem is true about the different bijective parts of the compression function of Blue Midnight Wish:

## Theorem 2.

1. When $H_{i-1}$ is fixed, $f_{0}\left(M_{i}, H_{i-1}\right)$ is a bijection.
2. For a given $H_{i-1}$, the function $f_{1}$ is a multipermutation between $M_{i}, Q_{a}$ and $Q_{b}$ i.e. for a given pair $\left(M_{i}, Q_{a}\right)$ it uniquely computes $Q_{b}$, for a given pair $\left(M_{i}, Q_{b}\right)$ it uniquely computes $Q_{a}$ and for a given pair $\left(Q_{a}, Q_{b}\right)$ it uniquely computes $M_{i}$.
3. When $Q_{b}$ and $M_{i}$ are fixed, $f_{2}\left(M_{i}, Q_{a}, Q_{b}\right)$ is a bijection.
4. When $Q_{b}$ and $Q_{a}$ are fixed, $f_{2}\left(M_{i}, Q_{a}, Q_{b}\right)$ is a bijection.

Proof. Item 1. This is a consequence of the non-singularity of the matrix $\mathbf{A}_{1}$ and the Lemma 3.
Item 2. (sketch) Let the pair $\left(Q_{a}, Q_{b}\right)$ be given. Then from equation $Q_{b}=f_{1}\left(M_{i}, H_{i-1}, Q_{a}\right)$ we can obtain an equation:

$$
\mathbf{B}\left(\operatorname{rot} M_{i}\right)=\text { Const }
$$

and since the matrix $\mathbf{B}$ is nonsingular we can find the unique solution $M_{i}$.
Item 3. (sketch) If $Q_{b}$ and $M_{i}$ are fixed then $H_{i}=f_{2}\left(M_{i}, Q_{a}, Q_{b}\right)$ can be rewritten as

$$
H_{i}=\left(L_{a}\left(Q_{b}\right) \oplus M_{i}\right)+\left(L_{b}\left(Q_{b}\right) \oplus Q_{a}\right)=\operatorname{Const}_{1}\left(Q_{b}, M_{i}\right)+\left(\operatorname{Const}_{2}\left(Q_{b}, M_{i}\right) \oplus Q_{a}\right),
$$

where $\operatorname{Const}_{1}\left(Q_{b}, M_{i}\right)$ and $\operatorname{Const}_{2}\left(Q_{b}, M_{i}\right)$ are expressions of the constants $Q_{b}$ and $M_{i}$. Here $H_{i}$ is a bijection of $Q_{a}$.

Item 4. (sketch) If $Q_{a}$ and $Q_{b}$ are fixed then $H_{i}=f_{2}\left(M_{i}, Q_{a}, Q_{b}\right)$ can be rewritten as

$$
H_{i}=\left(L_{a}\left(Q_{b}\right) \oplus M_{i}\right)+\left(L_{b}\left(Q_{b}\right) \oplus Q_{a}\right)=\left(\operatorname{Const}_{1}\left(Q_{a}, Q_{b}\right) \oplus M_{i}\right)+\operatorname{Const}_{2}\left(Q_{a}, Q_{b}\right)
$$

where $\operatorname{Const}_{1}\left(Q_{a}, Q_{b}\right)$ and $\operatorname{Const}_{2}\left(Q_{a}, Q_{b}\right)$ are expressions of the constants $Q_{a}$ and $Q_{b}$. Here $H_{i}$ is a bijection of $M_{i}$.

Note: Theorem 2 holds for every combination of ExpandRounds ${ }_{1}$ and ExpandRounds $s_{2}$ such that ExpandRounds ${ }_{1}+$ ExpandRounds $_{2}=16$.

### 3.7.2 Representation as a generalized PGV1 scheme with a weak block cipher

Preneel, Govaerts, and Vandewalle in [7] have located 12 secure schemes for constructing hash functions from block ciphers. Black et. al., [8] have proved (in an ideal cipher model) that those schemes are collision-resistant too.

The basic iterative relation for the scheme number 1 (PGV1) is:

$$
H_{i}=E\left(H_{i-1}, M_{i}\right) \oplus M_{i}
$$

where the notation $E(K, X)$ denotes a block cipher operation with a key $K$ and a plaintext $X$. The graphical representation of the scheme is given in Figure 3.1a.


Figure 3.1: a. The PGV1 one-way compression function, b. Generalized PGV1 one-way compression function where the feedback information of $M_{i}$ is combined with the ciphertext $E\left(H_{i-1}, M_{i}\right)$ not with a simple xor function $\oplus$ but with a more complex function $f_{2}$.

Theorem 3. BLUE MIDNIGHT WISH hash function can be expressed as a generalized PGV1 scheme.

Proof. (sketch) As a block cipher we can take the function $f_{0}\left(M_{i}, H_{i-1}\right) \equiv E\left(H_{i-1}, M_{i}\right)$. Then in a generalized version of PGV1 we can treat that the expression $E\left(H_{i-1}, M_{i}\right) \oplus M_{i}$ is represented in a generalized form as:

$$
H_{i}=f_{2}\left(M_{i}, H_{i-1}, E\left(H_{i-1}, M_{i}\right)\right)
$$

Note. The underlying block cipher $f_{0}$ used in this representation of Blue Midnight Wish is not ideal. Actually it is very weak. However, this deficiency of the block cipher used in BLUE MIDNIGHT WISH is compensated by the more complex feedback function, by the size of the block

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cipher output which is twice the size of the output of the hash function, and the bijective entanglements that are described in Theorem 2.

### 3.7.3 Monomial tests on the components used in Blue Midnight Wish

The monomial tests have been introduced several years ago by Filiol [9] to evaluate the statistical properties of symmetric ciphers. Later, Saarinen [10] proposed an extension of Filiol's ideas to a chosen IV statistical attack, called the "d-monomial test", and used it to find weaknesses in several proposed stream ciphers. In 2007, Englund, Johansson and Turan [11] generalized Saarinen's idea and proposed a framework for chosen IV statistical attacks using a polynomial description. Their basic idea is to select a subset of IV bits as variables, assuming all other IV values as well as the key being fixed. Then, by obtaining the algebraic normal form for such a function they were searching for some statistical deviations from ideal random Boolean function. A similar approach as that of Englund et al. is also described by O'Neil in [12].

In order to get a statistical measure of the deviation from ideal random Boolean function of the components that are used in Blue Midnight Wish we have defined NANT - A Normalized Average Number of Terms (monomials). NANT can be seen as a variant of Englund's monomial tests and it is defined in what follows.

Let $n \geq r \geq 1$ be integers and let $F:\{0,1\}^{n} \rightarrow\{0,1\}^{r}$ be a vector valued Boolean function. The vector valued function $F$ can be represented as an $r$-tuple of Boolean functions $F=$ $\left(F^{(1)}, F^{(2)}, \ldots, F^{(r)}\right)$, where $F^{(s)}:\{0,1\}^{n} \rightarrow\{0,1\}(s=1,2, \ldots, r)$, and the value of $F^{(s)}\left(x_{1}, \ldots, x_{n}\right)$ equals the value of the $s$-th component of $F\left(x_{1}, \ldots, x_{n}\right)$. The Boolean functions $F^{(s)}\left(x_{1}, \ldots, x_{n}\right)$ can be expressed in the Algebraic Normal Form (ANF) as polynomials with $n$ variables $x_{1}, \ldots, x_{n}$ of kind $a_{0} \oplus a_{1} x_{1} \oplus \cdots \oplus a_{n} x_{n} \oplus a_{1,2} x_{1} x_{2} \oplus \cdots \oplus a_{n-1, n} x_{n-1} x_{n} \oplus \cdots \oplus a_{1,2, \ldots, n} x_{1} x_{2} \ldots x_{n}$, where $a_{\lambda} \in$ $\{0,1\}$. Each ANF have up to $2^{n}$ terms (i.e. monomials), depending of the values of the coefficients $a_{\lambda}$. Denote by $L_{F^{(s)}}$ the number of terms in the ANF of the function $F^{(s)}$. Then the number of terms of the vector valued function $F$ is defined to be the number $L_{F}=\sum_{s=1}^{r} L_{F^{(s)}}$.

Definition 1. Let $F:\{0,1\}^{n} \rightarrow\{0,1\}^{r}$ be a vector valued Boolean function. For any $k \in\{1, \ldots, n\}$ and any assembly of $S$ subsets $\sigma_{j}=\left\{i_{1}, i_{2}, \ldots, i_{k}\right\} \subset\{0,1, \ldots, n-1\}$ chosen uniformly at random $(1 \leq j \leq S)$, let $F_{\sigma_{j}}$ denote the restriction of $F$ defined by

$$
F_{\sigma_{j}}\left(x_{1}, x_{2}, \ldots, x_{n}\right)=F\left(0, \ldots, 0, x_{i_{1}}, 0, \ldots, 0, x_{i_{2}}, 0, \ldots, 0, x_{i_{k}}, 0, \ldots, 0\right) .
$$

We define a random variable $\overline{L_{F}}$ - the Normalized Average Number of Terms (NANT) as:

$$
\overline{L_{F}}=\overline{L_{F}}(r, k)=\frac{1}{r} \cdot \frac{1}{2^{k-1}} \cdot \lim _{S \rightarrow \infty} \frac{1}{S} \sum_{j=1}^{S} L_{F_{\sigma_{j}}} .
$$

Since the subsets $\sigma_{j}$ are chosen uniformly at random, the average values of $L_{F_{\sigma_{j}^{(s)}}^{(s)}}(s=1,2, \ldots, r)$ are $2^{k-1}$ and the average value of $L_{F_{\sigma_{j}}}$ is $r 2^{k-1}$. Also, $L_{F_{\sigma_{j}}^{(s)}} \leq 2^{k}$. So, the following theorem is true:

Theorem 4. For any function $F:\{0,1\}^{n} \rightarrow\{0,1\}^{r}$ chosen uniformly at random from the set of all such functions, for any value of $r \geq 1$ and for any $k \in\{1, \ldots n\}$, it is true that

$$
0 \leq \overline{L_{F}} \leq 2
$$

and that the expected value is

$$
E X\left(\overline{L_{F}}\right)=1 .
$$

Note that if we want to apply the NANT measure on every bit of some function $F:\{0,1\}^{n} \rightarrow$ $\{0,1\}^{r}$ then instead of averaging on all $r$ coordinates we are taking that $r=1$ i.e., we have to apply the following formula:

$$
\overline{L_{F}}=\overline{L_{F}}(k)=\frac{1}{2^{k-1}} \cdot \lim _{S \rightarrow \infty} \frac{1}{S} \sum_{j=1}^{S} L_{F_{\sigma_{j}}} .
$$

We have measured NANT for every bit of $Q_{b}=\left(Q_{16}, \ldots, Q_{31}\right)$, the pair $(X L, X H)$ and the final chaining value $H_{i}=\left(H_{0}, \ldots, H_{15}\right)$.

By performing the NANT tests, we see that the component $Q_{16}$ is easily distinguishable from random Boolean function, while for all other variables in $Q_{b}$ the Boolean functions for every bit act as a random Boolean function. That is shown in Figure 3.2. For the two variables (XL, XH) which consist in total of 64 bits there are no significant deviations from the value 1.0 and that is shown in Figure 3.3. For the chaining variable $H_{i}$ there are also no significant deviations from the value 1.0 (Figure 3.4).

For digest sizes of 384 and 512 bits we have applied NANT tests on BMW512. The outcome of the NANT tests is similar with the case of BMW256. Namely, Boolean functions for the bits in $Q_{16}$ are easily distinguishable from random Boolean function, while for all other variables in $Q_{b}$ the Boolean functions for every bit act as a random Boolean function. That is shown in Figure 3.5. For the two variables $(X L, X H)$ which consist in total of 128 bits there are no significant deviations


Figure 3.2: NANT analysis on components of BMW256. Values of $\overline{L_{F}}$ for every bit (in total 512 bits) in $Q_{b}$.


Figure 3.3: NANT analysis on components of BMW256. Values of $\overline{L_{F}}$ for every bit in (XL,XH) (in total 64 bits).
from the value 1.0 and that is shown in Figure 3.6. For the chaining variable $H_{i}$ there are also no significant deviations from the value 1.0 (Figure 3.7).


Figure 3.4: NANT analysis on components of BMW256. Values of $\overline{L_{F}}$ for every bit (in total 512 bits) in $H_{i}$.


Figure 3.5: NANT analysis on components of BMW512. Values of $\overline{L_{F}}$ for every bit (in total 1024 bits) in $Q_{b}$.

So, we can say that although Blue Midnight Wish follows the well established and secure schemes for designing hash functions from block ciphers (PGV), its underlying block cipher is


Figure 3.6: NANT analysis on components of BMW512. Values of $\overline{L_{F}}$ for every bit in (XL, XH) (in total 128 bits).


Figure 3.7: NANT analysis on components of BMW512. Values of $\overline{L_{F}}$ for every bit (in total 1024 bits) in $H_{i}$.
a weak block cipher. But, does it make the overall design weak? We think that it does not make the overall hash function weak because of the following reasons:

1. The deficiency coming from the distinguishability of the first word (first 4 words) is compensated by the wide block size in Blue Midnight Wish which is 512 or 1024 bits long.
2. From the fifth word, all other words in $Q_{b}$ are not distinguishable from random 32-bit (64-bit) variables.
3. The feedback information is a complex function of the initial inputs to the block cipher and its output.

Additionally to the arguments described above, we want to justify our recommendation for the value ExpandRounds ${ }_{1}=2$. Namely, from the NANT analysis we have that the variable $Q_{17}$ which is obtained by the $\operatorname{expand}_{1}()$ function is already reaching the level of a random Boolean function. So, we can allow the rest of the variables in $Q_{b}$ (the variables $Q_{18}, \ldots, Q_{31}$ ) to be computed by the faster and less complex expansion function $\operatorname{expand}_{2}()$.

### 3.7.4 Infeasibility of finding collisions, preimages and second preimages

The design of BLUE MIDNIGHT WISH heavily uses combinations of bitwise operations of XORing, rotating and shifting (which can be seen as linear operations in $G F\left(2^{32}\right)$ and in $G F\left(2^{64}\right)$ ) and operations of addition and subtraction in $\mathbb{Z}_{2^{32}}$ or in $\mathbb{Z}_{2^{64}}$ (which are nonlinear operations in $G F\left(2^{32}\right)$ and in $G F\left(2^{64}\right)$ ). This strategy is combined with the mathematical property that BLUE MIDNIGHT WISH (without the final invocation of the compression function in the part "Finalize") can be represented as a generalized PGV1 scheme. The PGV1 design is second-preimage resistant and collision resistant. Moreover the final calling of the compression function with the constant CONSTfinal applies another robust one-way function on the result. Those are the reasons why we claim that BLUE Midnight Wish is a second-preimage resistant and collision resistant hash function.

Additionally, the diffusion characteristics of the Boolean functions $s_{i}(), i=0,1, \ldots, 5, f_{0}, f_{1}$ and $f_{2}$, the size of the chaining value being two times wider than the final message digest size, and the nonlinear expressions used in the functions $f_{0}, f_{1}$ and $f_{2}$, are the cornerstones of the BLUE Midnight Wish strength.

More specifically, the chaining part of Blue Midnight Wish - "The Double Pipe" is created by the folding function $f_{2}$ from three inputs, the current message block itself and the two nonlinear transformations of the message block and old chaining value ( $Q_{a}$ and $Q_{b}$ ). By having numerous bijective and multipermutation properties we can treat in some of those cases $Q_{a}$ and $Q_{b}$ as ciphertexts, created by non-linear block ciphers, but in a specific manner such that they are bijectively
tied together. The bijective entanglement, combined with the nonlinearity of the expressions in $f_{0}$, $f_{1}$ and $f_{2}$ gives us confidence that it is infeasible to find collisions, preimages or second preimages of Blue Midnight Wish. We think that it is hard to find a way to change consistently all three inputs (tied by non-linear bijective mappings) in such a way that these changes in 3-times wider input of the compression function $f_{2}$ will cancel each other or will lead to controllable changes.

The Blue Midnight Wish entanglement of the message, previous double pipe and the next double pipe is shown in Figure 2.2 for the compression function. The set of bijective entanglements in functions $f_{0}, f_{1}, f_{2}$ and in their inputs are the principle of defense of Blue Midnight Wish against collisions and preimages or their pseudo or near variants.

For instance when $M$ is fixed, the function $f_{0}$ ensures the (controlled) change in $Q_{a}$ and in AddElement as inputs (plaintext and the key) for the block cipher $f_{1}$. Let us suppose that the attacker is now able to solve the most difficult part of the scheme and suppose that he/she is now able to control the changes in the $f_{1}$ output (the ciphertext $Q_{b}$ ). Now, it is very improbable that he/she is also able to control the expression $f_{2}\left(\right.$ fixed $\left.M, Q_{a}, Q_{b}\right)$ thus making the attack noneffective. If we admit that the attacker is able to obtain even very near collision (for instance one bit difference) in the value $f_{2}$, the final invocation of the compression function will diffuse it into two hash values with approximately ideal Hamming distance (one half of the message digest).

Also, when $H$ is fixed, bijectivity of $f_{0}$ ensures the (controlled) change in $Q_{a}$ and in AddElement as inputs (plaintext and the key) for the block cipher $f_{1}$. The situation is now a little bit more complex then in previous case. But let us suppose that the attacker is now able to solve the most difficult part of the scheme and suppose that he/she is now able to control the changes in the $f_{1}$ output (the ciphertext $Q_{b}$ ). Now, it is very improbable that he/she is also able to control also the expression $f_{2}(M, Q a, Q b)$. And again, even if we admit that the attacker is able to obtain very near collision (for instance one bit difference) in the value $f_{2}$, the final invocation of the compression function will diffuse it into two hash values with approximately ideal Hamming distance (one half of the message digest).

The third case in this analysis is when $Q_{a}$ is fixed. The values of AddElement as a key for the fixed plaintext $Q_{a}$ are variable, and can be used to control the value $Q_{b}$, but we see that controlling all three values $M, H$ and $f_{2}\left(M, Q_{a}, Q_{b}\right)$ is very difficult and improbable. And as a last line of the defence we have again the final invocation of the compression function which will diffuse any near collision into two hashes with approximately ideal Hamming distance (one half of the message digest).

Another approach to attack Blue Midnight Wish can be for instance to fix the pair ( $Q_{a}, Q_{b}$ ).

But from the multipermutation property of $f_{1}$ we obtain unique value of $\operatorname{AddElement}(M, H)=$ $(\mathbf{B}(\operatorname{rot} M)+K) \oplus R O T L^{7}(H)$. Additionally we have another relationship between $M$ and $H$ coming from the $Q_{a}=f_{0}(M, H) \equiv \mathbf{A}_{2}\left(\mathbf{A}_{1} \cdot(M \oplus H)\right)+R O T L^{1}(H)$. So, we have two equations for two variables $M, H$. When we substitute from the first equation the variable $H$ into the second equation, we obtain one equation for one $16 * w$-bit variable $M$. This is in fact a system of $16 * w$ non-linear equations for $16 * w$ bit-variables (bits of $M$ ). This system is non-linear and complex (AXR - Addition, Xoring and Rotations) and there is no known effective method how to find a solution. Let us suppose that the attacker is able to solve it. Moreover let us suppose that he/she find out two different solutions $M$. Then he/she computes $H$ and thus has two different pairs $(M, H)$ leading to the same hash value. It is improbable that the attacker will obtain a solution of the form $(M, I V)$, so that at the best case what he/she obtains is a pseudo-collision.

There are more ways how to try to control some of the inputs, intermediate variables and output variables of BLUE Midnight Wish. But these variables are connected in such a way, that every time any change leads to guaranteed change or changes on several places in Blue Midnight WISH due to the bijective entanglement of all variables.

And again, as a last line of the defence we have the final invocation of the compression function which will diffuse any near collision of two hash values into hashes with approximately ideal Hamming distance (one half of the message digest).

### 3.7.5 Approximation of additions and subtractions with XORs

As mentioned in the previous subsection the compression function of Blue Midnight Wish uses bitwise operations of XORing, rotating and shifting (as linear operations in $G F\left(2^{32}\right)$ and in $G F\left(2^{64}\right)$ ) and operations of addition and subtraction in $\mathbb{Z}_{2^{32}}$ or in $\mathbb{Z}_{2^{64}}$ (as nonlinear operations in $G F\left(2^{32}\right)$ and in $\left.G F\left(2^{64}\right)\right)$.

A natural idea is to try to find values for which additions and subtractions behave as XORs. In such a case, one would have a completely linear system in the ring $\left(\mathbb{Z}_{2}^{n},+, \times\right)$ for which collisions, preimages and second preimages can easily be found. However, getting all the additions to behave as XORs is hard.

There are several significant works that are related with analysis of differential probabilities of operations that combine additions modulo $2^{n}$, XORs and left rotations. In 1993, Berson has made a differential cryptanalysis of addition modulo $2^{32}$ and applied it on MD5 [13]. In 2001, Lipmaa and Moriai, have constructed efficient algorithms for computing differential properties of addition

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modulo $2^{n}$ [14], and in 2004, Lipmaa, Wallén and Dumas have constructed a linear-time algorithm for computing the additive differential probability of exclusive-or [15].

All of these works are determining the additive differential probability of exclusive-or:

$$
\operatorname{Pr}[((x+\alpha) \oplus(y+\beta))-(x \oplus y)=\gamma]
$$

and the exclusive-or differential probability of addition:

$$
\operatorname{Pr}[((x \oplus \alpha)+(y \oplus \beta)) \oplus(x+y)=\gamma]
$$

where probability is computed for all pairs $(x, y) \in \mathbb{Z}_{2^{n}} \times \mathbb{Z}_{2^{n}}$ and for any predetermined triplet $(\alpha, \beta, \gamma) \in \mathbb{Z}_{2^{n}} \times \mathbb{Z}_{2^{n}} \times \mathbb{Z}_{2^{n}}$.

Recently Paul and Preneel [16] have successfully solved the problem of finding solutions in polynomial time of differential equations of addition with two variables $x$ and $y$ of type $(x+y) \oplus((x \oplus$ $\alpha)+(y \oplus \beta))=\gamma$ where $\alpha, \beta$ and $\gamma$ are constants. Someone can use their algorithm to try to attack Blue Midnight Wish. The problem is that their algorithm is for equations with two variables, and their strategy extended to solving systems of differential equations of addition with three or more variables has exponential complexity i.e. is of the order $O\left(2^{b \times k}\right)$ where $b$ is the bit length of the variables, and $k$ is the number of equations.

So, in the case of Blue Midnight Wish instead of a simple combination of two 32-bit (or 64-bit) variables once by additions then by xoring, we have a complex multivariate system of equations. In these equations both bitwise operations (XORing, shifting or rotation) and word-oriented operations (addition or subtraction) are mutually embedded one into the other. At the time of writing, we do not see how the results in [16] will help in finding solutions for the Blue Midnight Wish equations.

### 3.7.6 Cryptanalysis of a scaled down Blue Midnight Wish

Note: The results in this section were performed on the non-tweaked version of BLUE MIDNIGHT WISH. For the tweaked version we did not perform statistical tests on scaled down version of Blue Midnight Wish but having in mind that the tweaked version is slightly more complex and have additional final invocation of the compression function, we expect that the statistical properties of the scaled down version of the tweaked Blue Midnight Wish will be similar as the previous non-tweaked version.

In order to gain knowledge of how robust and sound the design of Blue Midnight Wish is, we analyzed a scaled down version of the algorithm. However, down-scaling of Blue Midnight

WISH require a different approach than that which is usually taken when the hash function has a big number of internal iterative steps which Blue Midnight Wish does not have. It has 16 expansion steps but those steps can not be reduced (since it will destroy the essence of the design working with different interconnected bijections). We have decided to down-scale the BLUE MIDNIGHT WISH by reducing the size of the word to 4 bits (corresponding to BMW224 and BMW256) and to 8 bits (corresponding to BMW384 and BMW512). In such a case we defined BMW28 (which has output of 7, 4 -bit words i.e. 28 bits), BMW32 (which has output of 8,4 -bit words i.e. 32 bits), BMW48 (which has output of 6, 8-bit words i.e. 48 bits) and BMW64 (which has output of 8, 8-bit words i.e. 64 bits). The summary is given in Table 3.1.

| Algorithm <br> abbreviation | Block size <br> $m$ (in bits) | Word size <br> $w$ (in bits) | Digest size <br> $n$ (in bits) |
| :---: | :---: | :---: | :---: |
| BMW28 | 64 | 4 | 28 |
| BMW32 | 64 | 4 | 32 |
| BMW48 | 128 | 8 | 48 |
| BMW64 | 128 | 8 | 64 |

Table 3.1: Basic properties of scaled-down variants of the BLUE MIDNIGHT Wish

In order for this down-scaling to be correct, we had to change (adapt) the logical functions used. In Table 3.2 we are listing the logical functions that we have used in the down-scaled version of Blue Midnight Wish. Note that we use the notation $\operatorname{ROTL}^{0}(x) \equiv x$ in order to show the consistency of the shape of logical functions in the scale-down function with the original construction of Blue Midnight Wish. All logical functions in the scaled-down hash function, similarly as in the original construction, are bijections in $G F\left(2^{w}\right)$ where $w=4,8,32,64$, is the size of the word on which these functions operate. The initial double-pipe value $H$ for this scaled-down functions has the value of the $w$ least significant bits of the double-pipe $H$ in the original design.

Having such a small hash outputs, it is easy to analyze and to find collisions for the compression functions of BMW28, BMW32 and BMW48 (but not so easy for BMW64 on our PC with 4GB RAM memory). The average number of calls to the compression functions before finding a collision in a hash of $n$ bits is given in the Table 3.3. Note that in the second column we give the average number $\mathcal{A}_{n}$ of calls to the compression function before finding a collision, and in the third column we give the theoretically expected number $\mathcal{T}_{n}$ of calls to the compression function for finding a collision.

Besides the attempts of finding collisions we have checked how good the randomness produced by the compression functions of these heavily scaled-down hash functions is. For doing that, for all four variants: BMW28, BMW32, BMW48 and BMW64, we have produced a 500 MBbytes file and

| BMW28/BMW32 | BMW48/BMW64 |
| :---: | :---: |
| $s_{0}(x)=S H R^{1}(x) \oplus S H L^{1}(x) \oplus \operatorname{ROTL}^{0}(x) \oplus \operatorname{ROTL}^{3}(x)$ | $s_{0}(x)=\operatorname{SHR}^{1}(x) \oplus \operatorname{SHL}^{1}(x) \oplus \operatorname{ROTL}^{3}(x) \oplus \operatorname{ROTL}^{4}(x)$ |
| $s_{1}(x)=\operatorname{SHR}^{1}(x) \oplus \operatorname{SHL}^{2}(x) \oplus \operatorname{ROTL}^{1}(x) \oplus \operatorname{ROTL}^{3}(x)$ | $s_{1}(x)=\operatorname{SHR}^{1}(x) \oplus \operatorname{SHL}^{2}(x) \oplus \operatorname{ROTL}^{1}(x) \oplus \operatorname{ROTL}^{6}(x)$ |
| $s_{2}(x)=\operatorname{SHR}^{2}(x) \oplus S H L^{1}(x) \oplus \operatorname{ROTL}^{3}(x) \oplus \operatorname{ROTL}^{0}(x)$ | $s_{2}(x)=\operatorname{SHR}^{2}(x) \oplus \operatorname{SHL}^{5}(x) \oplus \operatorname{ROTL}^{19}(x) \oplus \operatorname{ROTL}^{7}(x)$ |
| $s_{3}(x)=S H R^{2}(x) \oplus \operatorname{SHL}^{2}(x) \oplus \operatorname{ROTL}^{3}(x) \oplus \operatorname{ROTL}^{0}(x)$ | $s_{3}(x)=\operatorname{SHR}^{2}(x) \oplus S H L^{1}(x) \oplus \operatorname{ROTL}^{28}(x) \oplus \operatorname{ROTL}^{4}(x)$ |
| $s_{4}(x)=\operatorname{SHR}^{1}(x) \oplus x$ | $S_{4}(x)=\operatorname{SHR}^{1}(x) \oplus x$ |
| $s_{5}(x)=\operatorname{SHR}^{2}(x) \oplus x$ | $s_{5}(x)=\operatorname{SHR}^{2}(x) \oplus x$ |
| $r_{1}(x)=\operatorname{ROTL}^{1}(x)$ | $r_{1}(x)=\operatorname{ROTL}^{1}(x)$ |
| $r_{2}(x)=\operatorname{ROTL}^{2}(x)$ | $r_{2}(x)=\operatorname{ROTL}^{2}(x)$ |
| $r_{3}(x)=$ ROTL $^{3}(x)$ | $r_{3}(x)=\operatorname{ROTL}^{3}(x)$ |
| $r_{4}(x)=\operatorname{ROTL}^{0}(x)$ | $r_{4}(x)=\operatorname{ROTL}^{4}(x)$ |
| $r_{5}(x)=\operatorname{ROTL}^{1}(x)$ | $r_{5}(x)=\operatorname{ROTL}^{5}(x)$ |
| $r_{6}(x)=\operatorname{ROTL}^{2}(x)$ | $r_{6}(x)=\operatorname{ROTL}^{6}(x)$ |
| $r_{7}(x)=$ ROTL $^{3}(x)$ | $r_{7}(x)=\operatorname{ROTL}^{7}(x)$ |

Table 3.2: Logic functions used in scaled-down BLUE Midnight Wish

| $n$ | $\mathcal{A}_{n}$ | $\mathcal{T}_{n}$ |
| :---: | ---: | ---: |
| 28 | 20,108 | 20,480 |
| 32 | 84,511 | 81,920 |
| 48 | $21,469,868$ | $20,971,520$ |
| 64 | $/$ | $5,368,709,120$ |

Table 3.3: Finding collisions on scaled-down Blue Midnight Wish
examined its randomness by the "TestU01" - a C library for empirical testing of random number generators [17]. The methodology of producing those 500 MBbytes files was the following: We have represented the input message $M$ as a 64-bits (resp. 128 bits) counter with a starting value 1 increasing in steps of 1 . Then the counter $M$ was represented as 16, 4-bit (resp. 8 -bit) variables and we computed $h=$ Take_ $n \_$LS_bits $\left(f_{2}\left(M, f_{1}(M, H)\right)\right)$. The values $h$ were concatenated in order to build a 500 MBbytes file.

Report of TestU01 (applying two test batteries - Rabbit and the NIST FIPS-140-2) for BMW28 is given in Table 3.4 and for BMW32 in Table 3.5. From the reports it is clear that there are certain statistical tests that can distinguish the output of the compression function of BMW28 and BMW32 from an ideal source of randomness. Although the collision analysis for BMW28 and BMW32 are very close to those that are theoretically expected, intuitively it is expected that such heavily scaled-down instances of the original BLUE MIDNIGHT WISH will be distinguishable from an ideal source of uniformly distributed random bits.

However, if we consider that scaling down from 64-bit words to 8-bit words is a significant down-


Table 3.4: Summary of the TestU01 report for BMW28 (running the Rabbit and FIPS-140-2 battery)


Table 3.5: Summary of the TestU01 report for BMW32 (running the Rabbit and FIPS-140-2 battery)
scaling, we were surprised to see that BMW48 and BMW64 actually pass all statistical tests from Rabbit and FIPS-140-2 batteries. This clearly demonstrates the robustness of BlUE MiDNIGHT WISH design. TestU01 reports (applying again the test batteries - Rabbit and the NIST FIPS-140-2) are given in Table 3.6 and in Table 3.7. BMW48 and BMW64 pass all of these statistical tests.


Table 3.6: Summary of the TestU01 report for BMW48 (running the Rabbit and FIPS-140-2 battery)


Table 3.7: Summary of the TestU01 report for BMW64 (running the Rabbit and FIPS-140-2 battery)

### 3.8 Statements about security, support for applications, HMACs and randomized hashing

### 3.8.1 Security statement relating to the NIST requirement 4.A.

Security provided by BlUE Midnight Wish variants (BMW224, BMW256, BMW384, BMW512) in all applications (standards) is claimed to be the same or better than commensurate SHA-2 variants (SHA-224, SHA-256, SHA-384, SHA-512).

### 3.8.2 Statements relating to the NIST requirement 4.A.iii.

According to the analysis in previous sections we give a statement of the cryptographic strength of BlUE Midnight Wish against attacks for finding collisions, preimages, second preimages and resistance to length-extension attacks and multicollision attacks which is summarized in Table 3.8. Blue Midnight Wish of message digest size $n(n=224,256,384,512)$ meet the following security requirements:

- Collision resistance of approximately $\frac{n}{2}$ bits,
- Preimage resistance of approximately $n$ bits,
- Second-preimage resistance of approximately $n-k$ bits for any message shorter than $2^{k}$ bits,
- Resistance to length-extension attacks,
- Resistance to multicollision attacks, and
- Any $m$-bit hash function specified by taking a fixed subset of the Blue Midnight Wish's output bits meets the above requirements with $m$ replacing $n$.

| Algorithm <br> abbreviation | Digest size <br> $n$ (in bits) | Work factor for <br> finding collision | Work factor for <br> finding a preimage | Work factor for finding <br> a second preimage of a <br> message shorter than $2^{k}$ <br> bits | Resistance to <br> lenenth- <br> extension <br> attacks | Resistance to <br> multicollision <br> attacks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BMW224 | 224 | $\approx 2^{112}$ | $\approx 2^{224}$ | $\approx 2^{224-k}$ | Yes | Yes |
| BMW256 | 256 | $\approx 2^{128}$ | $\approx 2^{256}$ | $\approx 2^{256-k}$ | Yes | Yes |
| BMW384 | 384 | $\approx 2^{192}$ | $\approx 2^{384}$ | $\approx 2^{384-k}$ | Yes | Yes |
| BMW512 | 512 | $\approx 2^{256}$ | $\approx 2^{512}$ | $\approx 2^{512-k}$ | Yes | Yes |

Table 3.8: Cryptographic strength of the Blue Midnight Wish

### 3.8.3 Statement about the support of applications

All Blue Midnight Wish variants (BMW224, BMW256, BMW384, BMW512) support wide variety of cryptographic applications, including digital signatures (FIPS 186-2), key derivation (NIST Special Publication 800-56A), hash-based message authentication codes (FIPS 198), deterministic random bit generators (SP 800-90) in the same way as the corresponding SHA-2 variants (SHA224, SHA-256, SHA-384, SHA-512).

### 3.8.4 Statement about the special requirements

There are no special requirements when hash function Blue Midnight Wish is used to support HMAC, PRF and randomized hashing constructions. All Blue Midnight Wish variants (BMW224, BMW256, BMW384, BMW512) are used in these constructions (and in all appropriate standards) in the same way as the corresponding SHA-2 variants (SHA-224, SHA-256, SHA-384, SHA-512).

### 3.8.5 Support of HMAC

Blue Midnight Wish is an iterative cryptographic hash function. Thus, in combination with a shared secret key it can be used in the HMAC standard as it is defined in [18-20].

As the cryptographic strength of HMAC depends on the properties of the underlying hash function, and the conjectured cryptographic strength of Blue Midnight Wish is claimed in the Section 3.8.2, we can formally state that Blue Midnight Wish can be securely used with the HMAC. In what follows we are giving 4 examples for every digest size of $224,256,384$ and 512 bits.

BMW224-MAC Test Examples


## BMW256-MAC Test Examples

```
Key:
00010203 04050607 08090AOB OC0DOEOF 10111213 14151617 18191A1B 1C1D1E1F
20212223 24252627 28292A2B 2C2D2E2F 30313233 34353637 38393A3B 3C3D3E3F
Key_length: 64
Data:
'Sample #1'
Data_length: 9
HMAC:
B5F059FD 59189FA9 B4C0C11C 2B132C67 D89CBAE1 F116A2D2 A1539344 D8E2F938
Key:
30313233 34353637 38393A3B 3C3D3E3F 40414243
Key_length: 20
Data:
'Sample #2'
Data_length: 9
HMAC:
7B203B54 15EEF50E 6E64C1C7 58BD06D0 ED23D993 1F74F713 D49BD075 83251FFE
```

Key :
5051525354555657 58595A5B 5C5D5E5F 6061626364656667 68696A6B 6C6D6E6F 7071727374757677 78797A7B 7C7D7E7F 8081828384858687 88898A8B 8C8D8E8F 9091929394959697 98999A9B 9C9D9E9F A0A1A2A3 A4A5A6A7 A8A9AAAB ACADAEAF B0B1B2B3
Key_length: 100
Data:
'The successful verification of a MAC does not completely guarantee
that the accompanying message is authentic.'
Data_length: 110
HMAC:
6696C409 4F8D89BC EE17AF43 50DC4D3E 84A2E2CA 1A239DE8 C5B689F0 7FAF6248

Key:
5051525354555657 58595A5B 5C5D5E5F 6061626364656667 68696A6B 6C6D6E6F 7071727374757677 78797A7B 7C7D7E7F 8081828384858687 88898A8B 8C8D8E8F 9091929394959697 98999A9B 9C9D9E9F A0A1A2A3 A4A5A6A7 A8A9AAAB ACADAEAF B0B1B2B3
Key_length: 100
Data:
'The successful verification of a MAC does not completely guarantee that the accompanying message is authentic: there is a chance that a source with no knowledge of the key can present a purported MAC., Data_length: 200
HMAC:
F5C8A1F5 31FD09D1 F33845E7 05075A8C E5EEB29B 33EFF70B AE97B750 E3231383

## BMW384-MAC Test Examples

```
Key:
0001020304050607 08090AOBOCODOEOF 1011121314151617 18191A1B1C1D1E1F
2021222324252627 28292A2B2C2D2E2F 3031323334353637 38393A3B3C3D3E3F
Key_length: 64
Data:
'Sample #1'
Data_length: 9
HMAC:
E7BEAC8B685724D5 B625E79E007172DF 97FC85DB120DF5B7 52E618A676860EBB
73F46E70FAA0F084 937BFD6A21404913
Key:
3031323334353637 38393A3B3C3D3E3F 40414243
Key_length: 20
Data:
'Sample #2'
Data_length: 9
HMAC:
9E7DAF3407CB1BC0 CA3101F93A3D857B 44815D0C7203BC66 DE907C6C3DE7E322
E78A9072B285C97B EED23A85521F5EE7
```

Key
5051525354555657 58595A5B5C5D5E5F 6061626364656667 68696A6B6C6D6E6F 7071727374757677 78797A7B7C7D7E7F 8081828384858687 88898A8B8C8D8E8F 9091929394959697 98999A9B9C9D9E9F A0A1A2A3A4A5A6A7 A8A9AAABACADAEAF BOB1B2B350515253 5455565758595A5B 5C5D5E5F60616263 6465666768696A6B 6C6D6E6F70717273 7475767778797A7B 7C7D7E7F80818283 8485868788898A8B 8C8D8E8F90919293 9495969798999A9B 9C9D9E9FA0A1A2A3 A4A5A6A7A8A9AAAB ACADAEAFB0B1B2B3

Key_length: 200
Data:
'The successful verification of a MAC does not completely guarantee that the accompanying message is authentic.

Data_length: 110
HMAC:
515079D15A09C721 C63F3E1011DC7883 7D1362753377F861 FF34F9E884B84EA0 A60ADA03AF5FC724 870CCA900EC8E3B5

Key :
5051525354555657 58595A5B5C5D5E5F 6061626364656667 68696A6B6C6D6E6F 7071727374757677 78797A7B7C7D7E7F 8081828384858687 88898A8B8C8D8E8F 9091929394959697 98999A9B9C9D9E9F A0A1A2A3A4A5A6A7 A8A9AAABACADAEAF B0B1B2B3
Key_length: 100
Data:
'The successful verification of a MAC does not completely guarantee that the accompanying message is authentic: there is a chance that a source with no knowledge of the key can present a purported MAC.' Data_length: 200
HMAC:
9525578E38E7DD70 CB9FECB6DC72DEC0 388072FD3C63F6EC 733E26466DA7EEA2 3A5CD49C5B566D8E 730E30838F4C5563

BMW512-MAC Test Examples


### 3.8.6 BLUE MIDNIGHT WISH support of randomized hashing

Blue Midnight Wish can be used in the randomizing scheme proposed in [21, 22].

### 3.8.7 Resistance to SHA-2 attacks

Blue Midnight Wish is designed to have a security strength that is at least as good as the hash algorithms currently specified in FIPS 180-3 [23], and this security strength is achieved with significantly improved efficiency. Also, Blue Midnight Wish is designed so that a possibly successful attack on the SHA-2 hash functions is unlikely to be applicable to BlUE MidNight Wish.

Is it possible to use any idea from the attacks on SHA-2 (or any other hash function) also to BLUE Midnight Wish? Most ideas hardly use the concrete structure and operations of SHA-2. These concrete combinations of sums of variables, concrete operations, shifts, additions, xors, etc. are very important in any concrete attack. Any change, sometimes only a tiny change in the design
(the shift, xor instead of add, adding another variable) may require a massively changed attack to be mounted. The change in internal structure from SHA-2 to Blue Midnight Wish is huge. Different operations and combinations are used. All local collisions, neutral bits and so on, used in known attacks on SHA-2 (SHA-1) are thus ineffective and non-applicable, against BLUE MIDNIGHT WISH. No general method is known from the attacks on SHA-2, which would be applicable to Blue Midnight Wish.

The most important changes which have very strong effect in BLUE MIDNight Wish vs. SHA-2:
a. The use of bijections - it guarantees that any change on the input will give a change of the output. There are a lot of bijections in BLUE Midnight Wish and we found that it is difficult to cancel their influence.
b. The core of the bijections are non-linear transformations.
c. The use of bijections with good propagation characteristics - all linear and arithmetical bijections, used in BLUE MIDNIGHT WISH are designed to have precise (and good) propagation properties.
d. 16 summands (operands) are used in most operations. Unlike many other hash functions where in the compression functions they use basic mixing operation on 4,5 or 8 operands, Blue Midnight Wish in its core uses 16 operands (see the definition of the function $f_{1}$ ). It is very difficult to control many differences in operands of consecutive operations. Together with the bijective property of the transformations, we have a property that a single differential propagates very fast in the consecutive (iterative) core operations. From this, it follows that to break BLUE Midnight Wish it is necessary to develop new local collisions, new "rectangular relations", new neutral bits and even new strategies, rather than the old ones used in the analysis and the attacks on SHA-2 or on any other known hash function family.

Chapter 3: Design Rationale

# Estimated Computational Efficiency and Memory Requirements 

### 4.1 Speed of Blue Midnight Wish on NIST SHA-3 Reference Platform

We have developed and measured the performances of BLUE MIDNIGHT WISH on a platform with the following characteristics:

CPU: Intel Core 2 Duo,
Clock speed: 2.4 GHz ,
Memory: 4GB RAM,
Operating system: Windows Vista ${ }^{\text {TM }}$ Ultimate 64-bit (x64) Edition with Service Pack 1,
Compiler: $\operatorname{Intel}(\mathrm{R}) \mathrm{C}++$ 11.0.072.
We also tested it with the ANSI C compiler in the Microsoft Visual Studio 2005 Professional Edition, but that compiler was always giving worse results compared with the Intel compiler.

For measuring the speed of the hash function expressed as cycles/byte we have used the rdtsc () function and a modified version of a source code that was given to us by Dr. Brian Gladman from his optimized realization of SHA-2 hash function [24].

|  | Speed in cycles/byte for different lengths <br> (in bytes) of the digested message. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MD Size | 1 | 10 | 100 | 1000 | 10,000 | 100,000 |
| 224 | 3745.00 | 374.50 | 55.21 | 8.65 | 7.87 | 7.76 |
| 256 | 1165.00 | 115.30 | 17.17 | 8.66 | 7.87 | 7.76 |
| 384 | 8377.00 | 843.70 | 86.05 | 23.48 | 14.20 | 13.20 |
| 512 | 8497.00 | 848.50 | 86.05 | 23.48 | 14.17 | 13.20 |

Table 4.1: The performance of optimized 32-bit version of Blue Midnight Wish in machine cycles per data byte on Intel Core 2 Duo for different hash data lengths

|  | Speed in cycles/byte for different lengths <br> (in bytes) of the digested message. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MD Size | 1 | 10 | 100 | 1000 | 10,000 | 100,000 |
| 224 | 3601.00 | 362.50 | 52.81 | 8.37 | 7.59 | 7.50 |
| 256 | 1129.00 | 111.70 | 16.45 | 8.37 | 7.59 | 7.50 |
| 384 | 1177.00 | 115.30 | 11.65 | 4.57 | 3.95 | 3.90 |
| 512 | 1153.00 | 115.30 | 11.65 | 4.59 | 3.95 | 3.90 |

Table 4.2: The performance of optimized 64-bit version of Blue Midnight Wish in machine cycles per data byte on Intel Core 2 Duo for different hash data lengths

### 4.1.1 Speed of the Optimized 32-bit version of Blue Midnight Wish

In the Table 4.1 we are giving the speed of all four instances of Blue Midnight Wish for the optimized 32-bit version.

### 4.1.2 Speed of the Optimized 64-bit version of Blue Midnight Wish

In the Table 4.2 we are giving the speed of all four instances of Blue Midnight Wish for the optimized 64-bit version.

### 4.2 Memory requirements of Blue Midnight Wish on NIST SHA-3 Reference Platform

When processing the message block $M^{(i)}=\left(M_{0}^{(i)}, M_{1}^{(i)}, \ldots, M_{15}^{(i)}\right)$, we need memory for the current value of the double pipe $H^{(i-1)}=\left(H_{0}^{(i-1)}, H_{1}^{(i-1)}, \ldots, H_{15}^{(i-1)}\right)$, two auxiliary words XL and XH,
and value of the quadruple pipe $Q^{(i)}=\left(Q_{0}^{(i)}, Q_{1}^{(i)}, \ldots, Q_{31}^{(i)}\right)$.
The need of memory is thus:

- 16 words of $M^{(i)}$,
- 16 words of $H^{(i)}$,
- 2 words XL, XH,
- 32 words of $Q^{(i)}$.
which is in total 66 words. That means that BMW224 and BMW256 use 264 bytes and BMW384 and BMW512 use 528 bytes.


### 4.3 Estimates for efficiency and memory requirements on 8-bit processors

Daniel Otte has developed "AVR-Crypto-Lib" - a crypto library for 8-bit AVR microcontrollers [25] and implemented in C the non-tweaked version of BLUE MIDNIGHT WISH achieving the following results:

| Name | Size (Flash) <br> (bytes) | Cycles <br> (per byte) |
| :---: | :---: | :---: |
| BMW256 | 6024 | 67.56 |
| BMW512 | 15128 | 455.88 |

Table 4.3: Daniel Otte's results of the implementation of non-tweaked Blue Midnight Wish

In a private communication with Daniel Otte we got the information that he can reduce the size of the code in a range of $20 \%-50 \%$ and increase the speed in the same range if BLUE MIDNIGHT WISH is realized in assembler.

We estimate that the tweaked version of Blue Midnight Wish can achieve similar results (with approximately $10 \%$ penalty on the size and the speed).

Note: The rest of the claims given in this section are taken from the old documentation for the previous non-tweaked version of Blue Midnight Wish.

We have used 8-bit Atmel processors ATmega16 and ATmega64 to test the implementation and performance of the compression function of the two main representatives of the BLUE MIDNIGHT

Wish hash function: BMW256 and BMW512. We have used WinAVR - an open source software development tools for the Atmel AVR series of RISC microprocessors and for simulation we have used the AVR Studio v 4.14. In Table 4.4 we are giving the length of the produced executable code and the speed in number of cycles per byte.

| Name | Code size <br> (.text + data + bootloader) <br> in bytes | Speed <br> (cycles/byte) | 8-bit MCU |
| :---: | :---: | :---: | :---: |
| BMW224/256 | 10414 | 1369 | ATmega16 |
| BMW384/512 | 55810 | 2793 | ATmega64 |

Table 4.4: The size and the speed of code for the compression functions for BMW224/256 and BMW384/512

From the analysis of the produced executable code we can project that by direct assembler programming Blue Midnight Wish can be implemented in less than 8 Kbytes (BMW256) and in less than 32 KBytes (BMW512) but this claim will have to be confirmed in the forthcoming period during the NIST competition.

### 4.4 Estimates for a Compact Hardware Implementation

Note: The claims given in this section are taken from the old documentation for the previous nontweaked version of BLUE Midnight Wish. Additionally we can say that our estimates for the tweaked Blue Midnight Wish are the same as those for the non-tweaked version.

Our initial (non-optimized) VHDL implementation of BLUE MiDNIGHT WISH was done on Xilinx v3200efg1156-8 FPGA. In Table 4.5 we are giving obtained equivalent gate count and also estimates for the compact hardware implementation of the compression function of BLUE MIDNIGHT WISH. These estimates are based on the minimal memory requirements described in Section 4.2.

| Name | Obtained equivalent gate <br> count for Xilinx <br> v3200efg1156-8 | Estimated gate count for the <br> needed memory | Estimated gate <br> count for the <br> optimized <br> algorithm logic | Estimated minimal <br> total gate count |
| :---: | :---: | :---: | :---: | :---: |
| BMW224/256 | 44,983 | 12,672 | $\approx 4,000$ | $\approx 16,672$ |
| BMW384/512 | 84,515 | 25,344 | $\approx 6,000$ | $\approx 31,344$ |

Table 4.5: Obtained non-optimized gate count for the Xilinx v3200efg1156-8 FPGA, and estimated number of gate count for realization of the compression functions for BMW224/256 and BMW384/512

### 4.5 Internal Parallelizability of BLUE Midnight Wish

The design of Blue Midnight Wish allows very high level of parallelization in computation of its compression function. This parallelism can be achieved by using specifically designed hardware, and indeed with the advent of multicore CPUs, those parts can be computed in different cores in parallel. From the specification given below, we claim that Blue Midnight Wish can be computed after 20 "parallel" steps. Of course those 20 "parallel" steps have different hardware specification and different implementation specifics, but can serve as a general measure of the parallelizability of Blue Midnight Wish. The high level parallel specification of Blue Midnight WISH is as follows:

## Computing $f_{0}$

Step 1: Computation of all 16 parts of $W_{0}^{(i)}, W_{1}^{(i)}, \ldots, W_{15}^{(i)}$ can be done in parallel.
Step 2: Computing the values of all 16 parts of $Q_{a}$ can be done in parallel.

## Computing $f_{1}$

Step 1: It has 16 expansion steps and each step depends from the previous one. But every expansion step have an internal structure that can be parallelized, and a pipelined setup can compute parts from the next expansion steps that do not depend on the previous expansion value.

## Computing $f_{2}$

Step 1: This step can be computed together with the computation of Step 1 of the function $f_{1}$.
Step 2 (First half): Computation of the first 8 words $H_{0}^{(i)}, H_{1}^{(i)}, \ldots, H_{7}^{(i)}$ can be done in parallel.
Step 2 (Second half): Computation of the last 8 words $H_{8}^{(i)}, H_{9}^{(i)}, \ldots, H_{15}^{(i)}$ can be done in parallel.

## CHAPTER 5

## Statements

### 5.1 Statement by the Submitter

I, Svein Johan Knapskog, do hereby declare that, to the best of my knowledge, the practice of the algorithm, reference implementation, and optimized implementations that I have submitted, known as BLUE Midnight Wish may be covered by the following U.S. and/or foreign patents: NONE. I do hereby declare that I am aware of no patent applications that may cover the practice of my submitted algorithm, reference implementation or optimized implementations.

I do hereby understand that my submitted algorithm may not be selected for inclusion in the Secure Hash Standard. I also understand and agree that after the close of the submission period, my submission may not be withdrawn from public consideration for SHA-3. I further understand that I will not receive financial compensation from the U.S. Government for my submission. I certify that, to the best of my knowledge, I have fully disclosed all patents and patent applications relating to my algorithm. I also understand that the U.S. Government may, during the course of the lifetime of the SHS or during the FIPS public review process, modify the algorithm's specifications (e.g., to protect against a newly discovered vulnerability). Should my submission be selected for SHA-3, I hereby agree not to place any restrictions on the use of the algorithm, intending it to be available on a worldwide, non-exclusive, royalty-free basis.

I do hereby agree to provide the statements required by Sections 5.2 and 5.3, below, for any patent or patent application identified to cover the practice of my algorithm, reference implementation or optimized implementations and the right to use such implementations for the purposes of the SHA-3 evaluation process.

I understand that NIST will announce the selected algorithm(s) and proceed to publish the draft

FIPS for public comment. If my algorithm (or the derived algorithm) is not selected for SHA-3 (including those that are not selected for the second round of public evaluation), I understand that all rights, including use rights of the reference and optimized implementations, revert back to the submitter (and other owner[s, as appropriate). Additionally, should the U.S. Government not select my algorithm for SHA-3 at the time NIST ends the competition, all rights revert to the submitter (and other owners as appropriate).

Signed: Svein Johan Knapskog
Title:Prof.
Dated: 27 October 2008
Place: Trondheim, Norway

### 5.2 Statement by Patent (and Patent Application) Owner(s)

N/A

### 5.3 Statement by Reference/Optimized Implementations' Owner(s)

We, Danilo Gligoroski and Vlastimil Klima, are the owners of the submitted reference implementation and optimized implementations and hereby grant the U.S. Government and any interested party the right to use such implementations for the purposes of the SHA-3 evaluation process, notwithstanding that the implementations may be copyrighted.

Signed: Danilo Gligoroski
Title: Prof.
Dated: 27 October 2008
Place: Trondheim, Norway

Signed: Vlastimil Klima
Title: Mr.
Dated: 27 October 2008
Place: Prague, Czech Republic

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